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# Absolute interferometric shift-rotation method with pixel-level spatial frequency resolution

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ABSTRACT

Absolute interferometric testing method of shift-rotation in a Fizeau interferometer is effective in optical surface metrology with high accuracy and has been developed for decades. A pixel-level spatial frequency solution of interferometric shift-rotation method is presented in the manuscript. It requires a $90^{\circ}$ rotational measurement and at least two translational measurements with different translations in the $x$ and $y$ directions besides a measurement at an original confocal position ($0^{\circ}$). With the well-organized absolute procedures, the absolute surface deviation of the test and reference surface can be obtained accurately with pixel-level spatial frequency resolution. As a result, the absolute results of the method contain much more mid-to high spatial frequency information and it is useful to directly detect the defects (dusts, pits) of the test and reference surfaces. No orthogonal polynomials fitting (such as Zernike polynomials) and interpolations are required in the calculation, and the absolute results can be guaranteed with high accuracy. Experimental absolute results of flat surfaces are given.


## 1. Introduction

Absolute testing methods with Fizeau interferometers have been widely employed in optical surface metrology, to calibrate the reference surface deviation and improve the accuracy. The most traditional methods include two-sphere method, random-ball-averaging method and three-flat test method [1–3]. As another effective absolute testing method, shift-rotation method has been developed for so many years. And lots of valuable work has been done to promote the absolute method, such as the classical *N*-position method combined with rotationally symmetric Zernike polynomials fitting method which relies on the decomposition of surface deviation into rotationally asymmetric and symmetric components [4–7], the Zernike fitting method of full-aperture surface deviation [8] and other polynomials fitting method [9]. There are also other pixel-level spatial absolute methods. Reference. [10] describes an absolute shift method that requires only two translational measurements. But the method is effective only when the translations are both 1 pixel in the $x$ and $y$ directions. It is difficult to manipulate in practice. Recently, another more general and accurate absolute method has been proposed [11]. It relies on pixel-unknown variables of the surfaces of test and reference. For an aperture diameter of 901 pixels, there are over $1.2 \times 10^{6}$ unknown variables and $6 \times 10^{5}$ equations for each measurement of positions. It is a big challenge to solve the system of equations accurately and stably, and the data reduction that combines the interpolation of pixel-unknown variables and stitching algorithm is much more complicated.

In this paper, a comparative simple absolute method based on pixel-unknown variable is presented. It requires a measurement at an original confocal position ($0^{\circ}$), a $90^{\circ}$ rotational measurement and two translational measurements with different translations in the $x$ and $y$ directions. With those measurements, the absolute surface deviation of the test and reference surfaces can be obtained accurately with pixel-level spatial frequency resolution. As a result, the absolute results of the method contain much more mid-to high spatial frequency information and it is useful to directly detect the defects (dusts, pits) of the test and reference surfaces. No orthogonal polynomials fitting (such as Zernike polynomials) and interpolations are required in the calculation, and the absolute results can be guaranteed with high accuracy. The principle of the method is described in Section 2 in detail. Theoretical calculations and experimental absolute results of flat surfaces are presented in Section 3. Some discussions are given in Section 4.

## 2. Principle

Fizeau interferometers have been widely employed in highly accurate measurement of surface deviation. A simple sketch of a Fizeau interferometer to test flats is shown in Fig. 1 [10]. It is well

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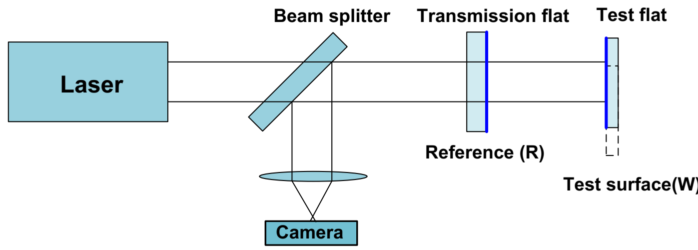


**Fig. 1.** Sketch of a Fizeau interferometer to test flats, reference surface deviation: *R*, test surface deviation: *W*.

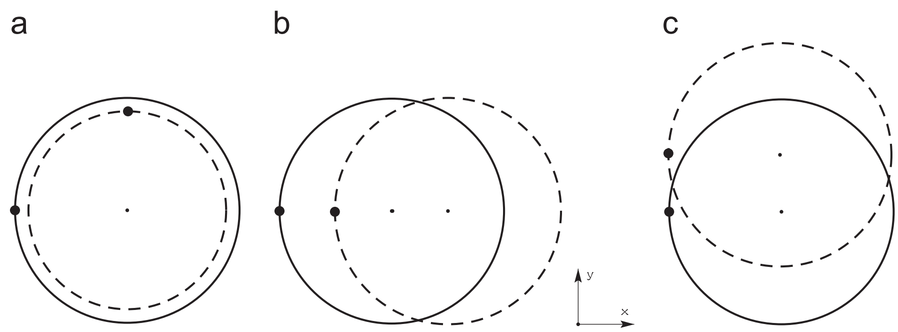


**Fig. 2.** Scheme of the absolute testing method of shift-rotation: (a) 90° rotational measurement; (b) translational measurement in the *x* direction and (c) translational measurement in the *y* direction.

known that a measurement result of a Fizeau interferometer can be expressed as the sum of the surface deviation of the test and reference surfaces [1,4,11]. The measurement at the original confocal position (0°) can be written as

$$T_1(x,y) = W(x,y) + R(x,y), \tag{1}$$

where $W(x,y)$and $R(x,y)$are the surface deviation of test and reference surfaces, respectively. The Cartesian coordinate $(x,y)$ is defined in the CCD.

Fig. 2 is a scheme of the absolute testing method [8]. It requires another 90° rotational measurement and two translational measurements with different translations in the $x$ and $y$ directions.

When the test surface is rotated with 90° around the optical axis as shown in Fig. 2(a), the measurement can be obtained as

$$T_2(x,y) = W_{90}(x,y) + R(x,y), \tag{2}$$

where $W_{90}(x,y)$ is the surface deviation of test surface when it is rotated by 90°; this means

$$W_{90}(\rho,\theta) = W(\rho,\theta+90), \tag{3}$$

$(\rho,\theta)$ is the corresponding polar coordinate.

Meanwhile, the absolute method requires two translational measurements with different translations in the $x$ and $y$ directions as shown in Figs. 2(b) and 1(c), respectively. The measurement results can be expressed as

$$T_3(x,y) = W(x+tx,y) + R(x,y), \tag{4}$$

$$T_4(x,y) = W(x,y+ty) + R(x,y), \tag{5}$$

where $tx$ and $ty$ are the translations of the test surface in the $x$ and $y$ directions, respectively.

Subtracting Eq. (1) from Eq. (2) to remove the reference surface deviation, we can get the equation

$$T_2(x,y) - T_1(x,y) = W_{90}(x,y) - W(x,y). \tag{6}$$

With Eqs. (4), (5) and (1), we can also get the equations

$$T_3(x,y) - T_1(x,y) = W(x+tx,y) - W(x,y). \tag{7}$$

$$T_4(x,y) - T_1(x,y) = W(x,y+ty) - W(x,y). \tag{8}$$

Eqs. (6)–(8) are the equations of the test surface deviation.

With the matrix method [8,10,11], Eq. (6) can be rewritten as

$$A_1X = B_1, \tag{9}$$

where $A_1$ is the coefficient matrix; $X$ is the vector of the pixel-unknown variables of the test surface deviation and $B_1$ is the corresponding vector of the effective pixel-point values of the measured results differences. Because the rotational angle is 90°, there will be no mismatch of the pixels in the subtraction as per Eq. (6). And there will be no interpolations of the pixel-unknown variables of the test surface deviation. As a result, the data reduction is much simpler.

With the same method, Eqs. (7) and (8) can be rewritten as

$$A_2X = B_2, \tag{10}$$

$$A_3X = B_3, \tag{11}$$

where $A_2$ and $A_3$ are the coefficient matrixes; $B_2$ and $B_3$ are the corresponding vectors of the effective pixel-point values differences. Combining those equations, we get

$$AX = B: \quad A = [A_1 \quad A_2 \quad A_3]^T,\ B = [B_1 \quad B_2 \quad B_3]^T \tag{12}$$

The above equation is over determined; the row number is much bigger than the column number. It is clear that the equation can be solved accurately with least-square algorithm only on the condition that the rank of $A$ is not less than the number of pixel-unknown variables subtracting 1. As known the interferometric measurement is relative; it is the difference of OPD (optical path difference) between the reference and test surfaces. After removing the reference surface deviation, the absolute surface deviation of an optical surface we get is actually relative to one point (pixel). We need to know only the relative relationship of an optical surface. If the rank of $A$ is not less than the number of pixel-unknown variables subtracting 1, the relative relationship is determined. Assuming that the grid size of the test surface deviation is $n$, this means

$$\mathrm{rank}(A) \geq n^2 - 1. \tag{13}$$