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The two-dimensional vector packing problem with general costs ☆

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ABSTRACT

The two-dimensional vector packing problem with general costs (2DVPP-GC) arises in logistics where shipping items of different weight and volume are packed into cartons before being transported by a courier company. In practice, the delivery cost of a carton of items is usually retrieved from a cost table. The costs may not preserve any known mathematical function since it could specify arbitrary price at any possible weight. Such a general pricing scheme meets a majority of real-world bin packing applications, where the price of delivery service is determined by many complicated and correlated factors. Compared to the classical bin packing problem and its variants, the 2DVPP-GC is more complex and challenging. To solve the 2DVPP-GC with minimizing the total cost, we propose a memetic algorithm to compute solutions of high quality. Fitness functions and improved operators are proposed to achieve effectiveness. Computational experiments on a variety of test instances show that the algorithm is competent to solve the 2DVPP-GC. In particular, optimal solutions are found in a second for all the test instances that have a known optimal solution.

1. Introduction

In courier logistics, shipping items are bundled and loaded into cartons before being transported by a courier company. Cartons with different weight are charged at different prices depending on the pricing scheme of the company. Considering a number of items to be delivered, an optimization problem of seeking a feasible packing plan that minimizes the total delivery cost is derived and faced by many manufacturers, retailers, and many other shippers that have the need of packaging and shipping goods. Among the different packing problems in real applications, the two dimensional vector packing problem with general costs (2DVPP-GC) is to solve the two dimensional vector packing problem with any cost structure that computes the delivery cost of a carton of items based on the total weight of the items. In the 2DVPP-GC, the general cost structure is of no necessity to preserve any well-known functions, such as linear or piecewise linear functions, but allows an arbitrary price at each possible weight.

A case study of the problem has been studied by Hu et al. [1]. It investigated the transportation operations of a manufacturer of children's apparel. The manufacturer had several production bases and hundreds of retail stores in the globe. It employed an express courier company under a long-term contract to ship manufactured products from a production base to a retail store. For a carton of items, where an item was formed by bundling articles of children's apparel, the transportation cost was charged by a courier company according to a cost structure, as shown in Fig. 1. The cost structure determined the amount of money that had to be paid by the manufacturer depending on the weight of the carton of items. The optimization of minimizing the transportation cost by finding a wise packing solution was concerned by the manufacturer. Such a problem is also faced by many other manufacturers. For instance, Zara, which leads in fast fashion industry and is well known for its agile supply chain, employs air courier service to distribute articles of apparel to countries outside Europe. To response quickly to demand, it prefers to reduce transit time at the expense of more transportation cost, which then becomes critical in its supply chain. As shown in Hu et al. [1], transportation cost can be reduced through improving packing solutions.

In the case study, only weight and volume attributes of items were considered in the packing because textile and garment products such as shirts are flexible and non-fragile. When a set of items is packed, the volume capacity and weight limit of a carton must not be exceeded. Fig. 1 depicts a cost structure that has been

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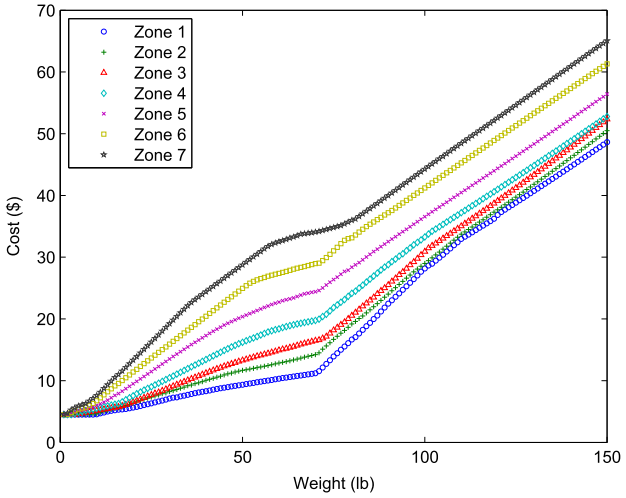

Fig. 1. Pricing schemes for 7 zones [1].

investigated in the case study. It contains 7 pricing schemes; each conveys a cost table that calculates the cost for delivering a carton from a source production base to a zone where the destination store is located. The delivery cost for the carton is determined by inputting its weight, which is rounded up to an integer if fractional. Generally, a cost table allows that the cost at each possible weight can be any arbitrary number. Such general costs that may not convey any linear or piecewise linear properties are common in real applications because pricing is affected by many complicated and correlated factors. Therefore, the problem of minimizing the transportation cost by packing all the shipping items into cartons is to solve the 2DVPP-GC.

Compared to the related bin packing problems in literature, the 2DVPP-GC is more challenging and complex due to the general costs involved. Hu et al. [1] opted to approximate the general costs into a piecewise linear cost function and solved the problem as the two dimensional vector packing problem with piecewise linear cost function (2DVPP-PLC). A solution to the 2DVPP-PLC is also feasible to the 2DVPP-GC. But the approximation may introduce considerable deviation from an optimal solution of the 2DVPP-GC. Our aim is to design a robust and effective method to solve the 2DVPP-GC directly to obtain high quality solutions without involving any approximation.

Through literature review (Section 2), we find that there is no prior work that is dedicated to the 2DVPP-GC. We then formally introduce the 2DVPP-GC and its mathematical formulation (Section 3). After discussing its complexity, we resort to design a memetic algorithm (Section 4) for the problem with introducing useful fitness functions and operators. Test instances of different sizes and various general costs are generated. We carry out comprehensive computational experiments on the test data (Section 5). The results show that the memetic algorithm is competent to solve the 2DVPP-GC and outperforms an algorithm which was first proposed for the 2DVPP-PLC in the literature.

2. Related literature

In the literature, there are some variants of the classical one-dimensional bin packing problem (BPP) [2,3] that consider to minimize the total cost of used bins. The costs are generated from the occupation of a bin or the transportation of items.

The occupation of a bin usually generates a fixed cost. The fixed cost appears in the BPP [4,5] and many of its variants including the variable-sized bin packing problem (VSBPP) [6–8], the two-dimensional vector packing problem (2DVPP) [9,10], the two-dimensional (rectangle) bin packing problem (2DBPP) [11–13], the three-dimensional (rectangle) bin packing problem (3DBPP) [14–16], and the multiple container loading cost minimization problem (MCLCMP) [17,18]. If the fixed cost for each bin is the same, the bin packing problem with minimizing the total cost is equivalent to the classical BPP where the number of used bins is minimized. The VSBPP extends the BPP by considering various bins of different sizes and fixed costs. The 2DVPP extends the BPP by adding a second attribute of the items and the bin and requiring that the total attribute of the items in a bin must not exceed the attribute capacity of the bin. In the 2DBPP, both the items and the bins are rectangles; the items must be orthogonally packed without overlaps. In the 3DBPP, the items and bins are boxes and containers, respectively; the boxes are orthogonally packed into the containers without overlaps. The MCLCMP combines the features of the 3DBPP and the VSBPP and considers various containers of different sizes and costs.

In logistics, the cost of a bin is the delivery cost of the items in the bin, as introduced in Hu et al. [1]. The pricing of delivery service is affected by many factors, such as weight, volume, manpower, equipment, and distance from source to destination. Therefore, the cost of a bin incurred in logistics is no longer a fixed cost, but more complicated. In the literature, concave cost functions [19,20], piecewise linear cost functions [1], and "general cost structures" [21,22] in the bin packing problems have been investigated. Li and Chen [19] and Leung and Li [20] investigated a variant of the BPP where the objective is to minimize the total cost of used bin and the cost of each bin is a concave function of bin utilization. Given the same weight limit of identical bins, the bin utilization also conveys the total weight of the items in the bin. That is, their approximation methods and results are also applicable to another bin packing variant where the cost of each bin is a concave function of the total weight of items. The "general cost structure" in the bin packing problem [21,22] is actually a concave function of the number of items in a bin instead of the total weight of the items in the bin. Like the problem studied by Li and Chen [19] and Leung and Li [20], the cost function considered must be monotone, non-decreasing, and concave. With such assumptions, analyses of the average-case or worst-case performance of simple heuristics were conducted. But it is difficult to apply them to the 2DVPP-GC, where arbitrary cost functions of the total weight of items could be.

In the literature, the 2DVPP-PLC [1] is close to our problem. In the 2DVPP-PLC, the cost of a bin is a piecewise linear cost function of the total weight of the items in the bin; the cost function can be neither concave nor convex. The 2DVPP-GC generalizes the 2DVPP-PLC by considering general costs. The 2DVPP-GC is more complex and challenging but more important, because it is concerned in more real-world bin packing applications. Hu et al. [1] studied an integer programming formulation and two heuristics for the 2DVPP-PLC. Their iterative local search heuristic can continuously improve solution quality but has defects on the slow convergence. None of the known optimal solutions for the instances with a known optimal solution were found by their heuristics. The memetic algorithm in this work is designed to overcome the defects and fast produce high quality solutions.

Heuristic approaches including genetic algorithms [7,23] and variable neighborhood search [5,24] have been successfully applied to solve bin packing problem and some variants. Haouari and Serairi [7] proposed a genetic algorithm for the VSBPP. In the VSBPP, the bins have different fixed costs. To optimally partition a sequence of items, Haouari and Serairi [7] introduced a graph representation and a shortest-path method. The partition method can be extended to be a sequence decoder for the 2DVPP-GC as well. Their operators of producing new individuals randomly

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