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A stochastic multi-period capacitated multiple allocation hub location problem: Formulation and inequalities[☆]

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ABSTRACT

This study focuses on the development of a modeling framework for multi-period stochastic capacitated multiple allocation hub location problems. We consider a planning horizon divided into several time periods. Uncertainty is assumed for the demands. The decisions to make concern the location of the hubs, their initial capacity, the capacity expansion of existing hubs and the transportation between origin–destination pairs. The goal is to minimize the total expected cost. For the situation in which uncertainty can be captured by a finite set of scenarios each occurring with some estimated probability we derive the extensive form of the deterministic equivalent. The resulting model is compact. However, it includes a set of binary variables that becomes too large for medium and large instances of the problem and thus hardly can it be tackled by a general optimization solver. For this reason, enhancements are proposed to the model making it possible to solve optimally instances that could not be solved using the initial model. This is confirmed by the computational tests performed using the well-known CAB data.

1. Introduction

Hub location problems have attracted much attention in the past decades due to their practical relevance. This can be observed in the recent book chapter by [11] as well as in the review papers by [3] and [10]. Nevertheless, these works and the extensive references lists therein show that most of the existing work focuses a static (single-period) problem, i.e., the full network is to be set up in a single step.

In this paper, we take into account the fact that establishing a hub network over time is often more realistic and even necessary (e.g., due to technical or budget constraints). Accordingly, we consider a planning horizon during which the hub location decisions as well as the other related decisions are to be implemented. Given that we are also considering capacitated hubs, this leads to a new possibility for increasing the operating capacity of the system: to expand the capacity at existing hubs. Such expansion can be done progressively over time and may render significant cost savings in comparison to locating new hubs. In addition to the need of considering time as an extra dimension in hub location, we observe that hardly is uncertainty avoidable in this type of decision making processes since some decisions, once implemented, have usually a long lasting effect (e.g., the construction of a new hub) and thus, they require data that often is not accurately known.

The above facts motivate the extension that we study in this work for the well-known capacitated multiple allocation hub location problem ([9,20]): we cast the problem as a stochastic multi-period decision making problem. In particular we consider a finite planning horizon divided into several time periods. The decisions to be made comprise (i) when and where to install new hubs, (ii) the initial capacity of new hubs, (iii) when and where to expand capacity for existing hubs, and (iv) how to route the flow between origin–destination (O–D) pairs in each period. The goal is to minimize the total cost for the entire planning horizon which contains several components, namely: setup cost for new hubs, setup cost for the initial capacity of new hubs, cost for expanding the capacity of existing hubs, cost for operating the hubs, and cost for routing the flow.

By considering the above mentioned decisions we are integrating the strategic and tactical/operational decision levels that often can be associated with a hub location problem. In fact, the

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decisions involving the location of hubs as well as their initial capacity have a strategic nature (they require time to be implemented, high investment costs and have limited reversibility); the expansion of existing hubs and the transportation of flows can be looked at as tactical/operational decisions (they require little time to be implemented).

The integration of location decisions with other types of decisions in a modeling framework has been done very frequently since early-on, researchers realized that suboptimal decisions would be obtained if the different decision levels were treated separately. As pointed out by [28], it was [26] who first realized that the location of facilities is often influenced by the transportation costs. Also in the context of "location plus transportation decisions", Salhi and Nagy [33] found out that by combining location (strategic) and routing (tactical/operational) decisions in a single modeling framework, leads to solutions that "could decrease the total cost over a long planning horizon, within which routes are allowed to change". In the case of hub location problems (again location and transportation decisions are combined) we can observe that since the early seminal works, the scientific community has combined both types of decisions. In this work we do the same.

We assume that a non-hub node can be allocated (and thus ship traffic) to more than one hub. Furthermore, the allocations can change from one period to another. Additionally, we impose that all flow traversing links between hubs has been already processed at some hub (e.g., in the case of mail, we assume that when it traverses a link between two hubs it has already been sorted out). This assures that the discount cost typically associated with the traffic between hubs does not apply to unprocessed flow. This is an aspect mentioned by [7] but fully explored only by [5].

We assume that the hubs' capacities are modular, that is, the operating capacity of a hub at some point in time is determined by the modules installed so far in that hub. Modular capacities are relevant when the capacity of a facility cannot be expanded continuously (e.g., interfaces in a transportation system or sorting lines in a mail distribution system). Nevertheless, this is a feature that again has been scarcely considered in the hub location literature. Some works dealing with it are those by [14,17] and [5].

Finally, another relevant feature that we consider is uncertainty associated with the demands originated in the different periods of the planning horizon. We consider that such uncertainty can be captured by a finite set of scenarios, each with some occurrence probability known (e.g., estimated using historical data) in advance. A risk neutral attitude is assumed for the decision maker. This means that the current value of future assets will be captured by expected values.

To the best of the authors' knowledge, no work has been published within the area of hub location casting stochasticity in a multi-period setting even though some works can be found dealing with these aspects separately: the papers by [8,12] and [5] study multi-period hub location problems; stochastic hub location problems have been investigated by [4,13,27,34,35] and [32].

The problem we are investigating is formulated as a two-stage stochastic programming problem with first-stage integer variables and both integer and continuous variables in the second stage. The first stage problem involves the here-and-now strategic decisions and consists of defining a plan for locating the hubs and setting up their initial capacity for the entire planning horizon; the second stage problem captures the tactical/operational decisions, i.e., capacity expansion of existing hubs and flow routing. Note that in both stages a multi-period plan is being defined. The application of a two-stage stochastic modeling framework to a (stochastic) multi-period problem is not new. The works by [2] and [1] are worth mentioning in this context.

A capacitated multiple allocation hub location problem featuring the aspects we are considering leads to large-scale mixed-integer linear optimization models that, as we will show, can be enhanced by valid inequalities that can significantly improve the polyhedral description of the feasibility set. This aspect is of particular relevance for practitioners who are often not well acquainted with more sophisticated solution techniques for combinatorial optimization problems. Additionally, this improved description is of great relevance for obtaining sharp lower bounds on the optimal value of the problem, which, in turn is very important for evaluating approximate approaches developed for obtaining feasible solutions to the problems. It is worth noticing that the development of valid inequalities for the feasibility set of stochastic integer and mixed-integer programs has been scarcely focused in the literature. The paper by [23] provides an excellent but rare contribution to the topic.

The problem we are investigating represents a first step in terms of including stochasticity in multi-period hub location and thus to understand the resulting extra difficulty that emerges from that. This is the reason for neither considering the possibility of closing hubs nor to decrease their capacity and thus to keep our analysis more focused. The inclusion of such decisions would considerably increase the difficulty to the problem as we can observe in the paper by [21]. Naturally, the success of the study we are presenting in the current work, opens new research directions namely for situations in which the above decisions are included.

The remainder of this paper is organized as follows: in Section 2 we propose the modeling framework for our problem and we write the deterministic equivalent in its extensive form. The problem is made more tractable from the perspective of using a general purpose solver in Section 3 where we propose several sets of valid inequalities for enhancing the above mentioned model. In Section 4 we present the computational tests performed using an off-the-shelf solver for solving the models proposed in Sections 2 and 3. The paper ends with a summary of the work done, some conclusions drawn from it, and some directions for further research.

2. Problem formulation

In this section we introduce an optimization model for the stochastic multi-period capacitated multiple allocation hub location problem. In order to make the model easier to read, we start by presenting a deterministic version of the problem and afterward we extend it to the stochastic setting.

2.1. A deterministic multi-period capacitated multiple allocation hub location problem

We consider as a starting point the well-known optimization model proposed by [20] for the (static/single-period) multiple allocation capacitated hub location problem. Afterward, we include and extend some features considered by [5] for a multi-period hub network design problem with modular capacities. As a result, we obtain a deterministic multi-period capacitated multiple allocation hub location problem which is at the core of our new development. We start itemizing the assumptions that we make.

- The planning horizon is finite and divided into several time periods.
- The potential hubs define a subset of the initial set of nodes.
- At the beginning of each time period it is possible to open new hubs. In that case it is necessary to decide the number of modules that define the initial capacity of the hub. In the beginning of the following periods it is possible to install additional modules.
- A limit exists for the number of modules that can be installed at each location.