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Best-worst multi-criteria decision-making method

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ABSTRACT

In this paper, a new method, called best-worst method (BWM) is proposed to solve multi-criteria decision-making (MCDM) problems. In an MCDM problem, a number of alternatives are evaluated with respect to a number of criteria in order to select the best alternative(s). According to BWM, the best (e.g. most desirable, most important) and the worst (e.g. least desirable, least important) criteria are identified first by the decision-maker. Pairwise comparisons are then conducted between each of these two criteria (best and worst) and the other criteria. A maximin problem is then formulated and solved to determine the weights of different criteria. The weights of the alternatives with respect to different criteria are obtained using the same process. The final scores of the alternatives are derived by aggregating the weights from different sets of criteria and alternatives, based on which the best alternative is selected. A consistency ratio is proposed for the BWM to check the reliability of the comparisons. To illustrate the proposed method and evaluate its performance, we used some numerical examples and a real-word decision-making problem (mobile phone selection). For the purpose of comparison, we chose AHP (analytic hierarchy process), which is also a pairwise comparison-based method. Statistical results show that BWM performs significantly better than AHP with respect to the consistency ratio, and the other evaluation criteria: minimum violation, total deviation, and conformity. The salient features of the proposed method, compared to the existing MCDM methods, are: (1) it requires less comparison data; (2) it leads to more consistent comparisons, which means that it produces more reliable results.

1. Introduction

Multi-criteria decision-making (MCDM) is a very important branch of decision-making theory. MCDM problems are generally divided into two classes with respect to the solution space of the problem: continuous and discrete. To handle continuous problems, multi-objective decision-making (MODM) methods are used. Discrete problems, on the other hand, are solved using multi-attribute decision-making (MADM) methods, which are the focus of this paper. In existing literature, however, MCDM is commonly used to describe the discrete MCDM, which is why we also use MCDM in this paper.

A 'discrete MCDM' problem (hereafter, for the sake of simplicity and in line with common practice, MCDM) is generally shown as a matrix, as follows:

$$A = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{pmatrix} \end{array} \quad (1)$$

where $\{a_1, a_2, \ldots, a_m\}$ is a set of feasible alternatives (actions, stimuli), $\{c_1, c_2, \ldots, c_n\}$ is a set of decision-making criteria, and p_{ij} is the score of alternative i with respect to criterion j. The goal is to select the best (e.g. most desirable, most important) alternative, in other words an alternative with the best overall value. The overall value of alternative i, V_i can be obtained using various methods. In a general form, if we assign weight w_j ($w_j \geq 0$, $\sum w_j = 1$) to criterion j, then V_i can be obtained using a simple additive weighted value function [1], which is the underlying model for most MCDM methods, as follows:

$$V_i = \sum_{j=1}^{n} w_j p_{ij} \quad (2)$$

What is very important here, and which has been the impetus of introduction of several MCDM methods during the last decades, is the way in which the weights of the criteria or vector $w = \{w_1, w_2, \ldots, w_n\}$ is obtained.

Over the last decades, several MCDM methods have been proposed, the most popular of which are AHP (Analytic Hierarchy Process) [2–4], ANP (Analytic Network Process) [5], TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [6–10], ELECTRE (ELimination Et Choix Traduisant la REalité) (ELimination and Choice Expressing REality) [11–13], VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) [14], and PROMETHEE (Preference Ranking

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Organization METHod for Enrichment Evaluations) [15–18]. For some recent developments we refer to the superiority and inferiority ranking (SIR) method [19], step-wise weight assessment ratio analysis (SWARA) [20], subjective weighting method using continuous interval scale [21], multi-attribute evaluation using imprecise weight estimates (IMP) [22] among others. For the study and comparison of different MCDM methods we refer to [23–27].

Pairwise comparison method which was first introduced by Thurstone [28] under the *law of comparative judgment* which is in fact implied in Weber's law and in Fechner's law [29] is an structured way to make the decision matrix. Pairwise comparisons (which are provided by expert or a team of experts) are used to show the relative preferences of m stimuli or actions in situations where it is unfeasible or meaningless to provide score estimates for the stimuli or actions with respect to criteria. For instance, underlying on ratio-scaling method [30,31], in the AHP, the weights are derived from pairwise comparisons of the criteria and the scores are derived from pairwise comparisons of the alternatives against the criteria, after which a similar function like (2) is used to calculate the overall value of alternatives. The very significant challenge to the pairwise comparison method comes from the lack of consistency of the pairwise comparison matrices which usually occurs in practice [32].

The pairwise comparison matrix $A=(a_{ij})_{n\times n}$ is considered to be perfectly consistent if, for each i and j, $a_{ik}\times a_{kj}=a_{ij}$. Unfortunately, however, for several reasons (for instance lack of concentration) there are recurring inconsistencies in pairwise comparison matrices [33]. When a comparison matrix is inconsistent, the recommended course of action is to revise the comparison such that the comparison matrix becomes consistent. Although this is a very common approach, it has been shown not to be successful [34]. In our opinion, the main cause of the inconsistencies mentioned above is in the unstructured way comparisons are executed by pairwise comparison-based methods. The main contribution of this paper is to propose a new multi-criteria decision-making method that derives the weights based on pairwise comparisons in a different way compared to the existing MCDM methods. We will demonstrate that the approach proposed in this paper uses less comparison data compared to the other MCDM methods, and that it remedies the inconsistency that characterizes the kind of pairwise comparisons in question.

The remainder of this paper is organized as follows. In Section 2, a new MCDM method (BWM) is proposed. In Section 3, the BWM is applied to a real-world problem, and it is comprehensively compared to the AHP considering several evaluation criteria. The conclusions and suggestions for future research are presented in Section 4.

2. Best-worst method (BWM)

Suppose we have n criteria and we want to execute a pairwise comparison these criteria using a 1/9 to 9 scale[1]. The resulting matrix would be

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \tag{3}$$

where a_{ij}shows the relative preference of criterion i to criterion j. $a_{ij}=1$ shows that i and j are of the same importance. $a_{ij}>1$ shows that i is more important than j with $a_{ij}=9$ showing the extreme importance of i to j. The importance of j to i is shown by a_{ji}. In order for matrix A to be reciprocal, it is required that $a_{ij}=1/a_{ji}$ and $a_{ii}=1$, for all i and j. Considering the reciprocal property of matrix A, in order to obtain a completed matrix A, it is necessary to have $n(n-1)/2$ pairwise comparisons. The pairwise comparison matrix A is considered to be perfectly consistent if:

$$a_{ik}\times a_{kj}=a_{ij}, \quad \forall i,j \tag{4}$$

Here we try to make a better understanding of the so-called pairwise comparison, which, in fact, makes the foundation of our proposed method (BWM).

When executing a pairwise comparison a_{ij}, the decision-maker expresses both the *direction* and the *strength* of the preference i over j. In most situations, the decision-maker has no problem in expressing the direction. However, expressing the strength of the preference is a difficult task that is almost the main source of inconsistency. To understand this important issue better, we use a visual illustration (Figs. 1 and 2).

Comparing tree A with the other trees in Fig. 1, it may be easy to determine that A is shorter than B and taller than the other trees (direction). However, assigning a number to express the level of relative tallness (strength) is more difficult. In fact, when one wants to assign a number to show one's judgment with regard to comparing A and B, one also keeps in mind the relationships between these two and some others. For example, suppose one

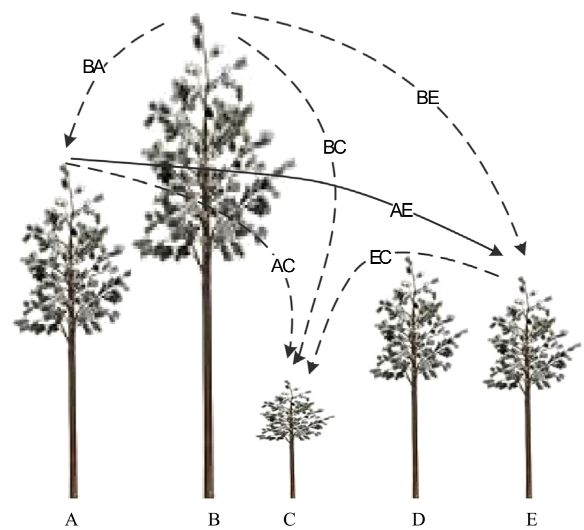

Fig. 1. A comparison example: the preference of A over E.

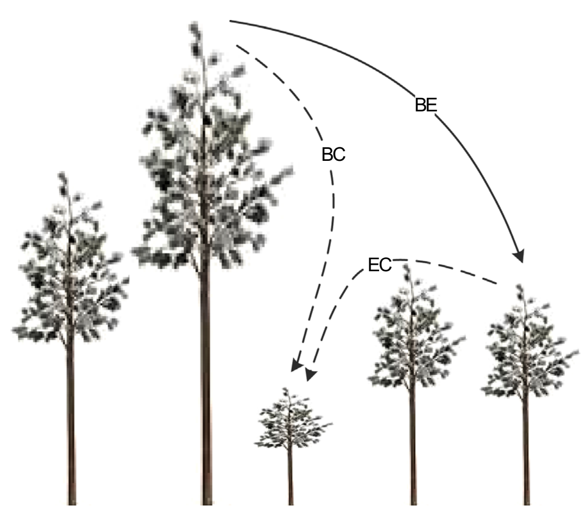

Fig. 2. A comparison example: the preference of B over E.

[1] It is possible to use other scales like 0.1 to 1.0, or 1 to 100.