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Centralized resource allocation based on efficiency analysis for step-by-step improvement paths

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ABSTRACT

The existing centralized resource allocation models often assume that all of the DMUs are efficient after resource allocation. For the DMU with a very low efficiency score, it means the dramatic reduction of the resources, which can cause the organizational resistance. In addition, in reality, it is particularly difficult for the DMUs to achieve their target efficiencies in a single step, especially when they are far from the efficient frontier. Thus, gradual progress towards benchmarking targets is gaining importance. In this paper, we present a new approach for resource allocation based on efficiency analysis under a centralized decision-making environment. Through our approach, the central decision-maker can obtain a sequence of intermediate benchmark targets based on efficiency analysis, which provide a level-wise improvement path to direct the DMUs to reach their ultimate targets on the efficient frontier in an implementable and realistic manner. Numerical examples are presented to illustrate the application procedure of the proposed approach.

1. Introduction

Data envelopment analysis (DEA) is an approach for measuring the performance of decision making units (DMUs) that convert multiple inputs into multiple outputs. Since the development of DEA by Charnes et al. [5], it has been applied to problems in many areas, and a number of theoretical additions have been made. In the traditional DEA models, each DMU is projected independently either input-oriented or output-oriented. However, the DMUs being assessed may fall under the umbrella of a centralized decision-maker who can be more interested in optimizing the operations of all of the units globally. In particular, when considering input orientation, the central decision-maker may be interested in simultaneously reducing the total inputs by all of the units jointly, rather than reducing the inputs of the individual DMU independently.

In recent years, DEA has been used increasingly for resource allocation under such a centralized environment. A number of papers about DEA applications for resource allocation in a centralized environment have been published [15,3,19,8,24]. Lozano and Villa [15] present two centralized data envelopment analysis (CDEA) models to reduce the total quantity of resources consumed by all units in an organization rather than considering the consumption of each unit separately. Asmild et al. [3] extended the concepts of Lozano and Villa [15] and modified them to only consider adjustments of previously inefficient units. They proposed the centralized resource allocation BCC model and applied it to a public service organization. Mar-Molinero et al. [19] attempted to interpret the Lozano and Villa [15] model and to show that it can be substantially simplified, which makes the model easier to implement in many situations. Some centralized resource allocation models based on output orientation include Lozano et al. [16,17], Du et al. [7], Hosseinzade et al. [11], Lozano and Villa [18], Amirteimoori and Emrouznejad [1], and Fang and Li [9].

All of the studies outlined above assume that each DMU is projected to be efficient after its resource allocation. Normally, the internal structure of a DMU should not change dramatically in the short term. If a rapid change policy is conducted, it can cause organizational resistance and reduced performance [2,10,25]. A recent example is the dismissal of the regular employees in an airport organizational setting in Taiwan [25]. In addition, as in the case of Digital Agenda goals for Europe, where the specific targets are long term and/or require significant improvements, it is particularly difficult to achieve immediately or within a specified timeframe [21]. Moore (2004) studied this problem and found that it is often the case that only 50% of the targets are achieved. Thus, it is practically infeasible for the inefficient DMUs to achieve their target efficiencies in a single step, especially when they are far from the efficient frontier [4,13,23]. One way to address successive performance improvement is found in the literature under the term stepwise benchmarking [6,21]. For example,

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based on the idea of the context-dependent DEA proposed by Seiford and Zhu [22], several researchers [4,13,23] constructed multiple efficient layers to provide level-wise benchmarking paths for inefficient units to reach the targets on the efficient frontier. In light of regular and contracted manpower constraints under different strategies, Yu et al. [25] developed a fit change approach to reallocate human resources for 18 Taiwan airports under three different human resource reallocation policies (denoted as short-term policy, middle-term policy, and long-term policy, respectively). However, the change of their efficiency scores is not considered during the resource allocation. Further, Korhonen and Syrjänen [12] developed a general approach for resource allocation in the centralized decision-making environment. The aim is to maximize the total amount of outputs of all units simultaneously by allocating available resources to them based on the assumption that the inefficiency scores of all unit cannot decrease. Nasrabadi et al. [20] defined the overall performance as a convex combination of the ratio of the efficiencies before and after the resource allocation. Assuming that the efficiency score of each DMU cannot be decreased, they developed a novel model to allocate the available resources or to reallocate the current resources for achieving the best overall performance of the system.

In this paper, we proposed a new centralized resource model, which can specify the efficiency score of each DMU after its resource allocation. In particular, in contrast to the previously presented models in a centralized environment, where benchmarking allows for the identification of targets for improvement, it does not prescribe any level-wise improvement path and so it becomes intellectually challenging for managers to make proper improvement strategies [23]. Through our proposed approach the central decision-maker can obtain a sequence of intermediate benchmark targets, which provide a level-wise improvement path to direct the DMUs to reach their ultimate targets on the efficient frontier in an implementable and realistic way.

The rest of the paper is organized as follows. In Section 2, we briefly review the centralized resource allocation models proposed by Lozano and Villa [15]. A centralized resource allocation model based on efficiency analysis is proposed in Section 3. Section 4 applies the proposed approach to a real dataset. Concluding remarks are presented in Section 5.

2. Preliminaries

Working in the usual DEA framework, let us consider a set A of n DMUs, with each DMU consuming m inputs to generate s outputs. The input and output matrices are denoted by $\mathbf{X} \in R^{m\times n}$ and $\mathbf{Y} \in R^{s\times n}$, respectively. For each DMU$_j$ $(j=1,\cdots,n)$, $(\mathbf{x}_j,\mathbf{y}_j)$ represents the observed input and output vectors, in which $\mathbf{x}_j=(x_{1j},\cdots,x_{mj})$ and $\mathbf{y}_j=(y_{1j},\cdots,y_{sj})$. The input-oriented DEA model (BCC-I) for evaluating the DMU$_o$ can be represented as follows:

$$\begin{aligned} & \text{Min } 1-z \\ \text{s.t. } & \mathbf{X}\lambda \le (1-z)\mathbf{x}_o \\ & \mathbf{Y}\lambda \ge \mathbf{y}_o \\ & \mathbf{1}^T\lambda = 1 \end{aligned} \tag{1}$$

where $\mathbf{1}=[1,\cdots,1]^T \in R^n$. $\lambda=[\lambda_1,\lambda_2,\cdots,\lambda_n]^T \in R^n_+$ represents the non-negative vector used to construct the efficient frontier by a convex combination of n DMUs. The inefficiency score z_o ranges between zero and one, with DMU$_o$ being considered relatively efficient if it receives a score of zero.

Assuming that all DMUs are under the control of the centralized decision-maker, the centralized DM aims to minimize the total input consumption by all DMUs in the organization and still achieve the same total outputs. The centralized resource allocation model proposed by Lozano and Villa [15] is:

$$\begin{aligned} & \text{Min } 1-z \\ \text{s.t. } & \sum_{i=1}^{n} \mathbf{X}\eta_i \le (1-z)\mathbf{X}\mathbf{1} \\ & \sum_{i=1}^{n} \mathbf{Y}\eta_i \ge \mathbf{Y}\mathbf{1} \\ & \mathbf{1}^T\eta_i = 1 \qquad i=1,\cdots,n \\ & \eta_i \ge 0 \qquad i=1,\cdots,n \end{aligned} \tag{2}$$

where $\mathbf{1}=[1,\cdots,1]^T \in R^n$ and $\eta_i=[\eta_{i1},\eta_{i2},\cdots,\eta_{in}]^T \in R^n_+$. Once the above model (2) is solved, the corresponding vector set of the input and output targets for the DMUs are as follows:

$$\hat{\mathbf{x}}_i = \mathbf{X}\eta_i \quad i=1,\cdots,n \tag{3}$$

$$\hat{\mathbf{y}}_i = \mathbf{Y}\eta_i \quad i=1,\cdots,n \tag{4}$$

It is proven that the projected point $(\hat{\mathbf{x}}_i,\hat{\mathbf{y}}_i)$ for DMU$_i$ after resource allocation is technically efficient. In the above model (2), all DMUs are projected onto the technically efficient frontier. However, the dramatic change of the internal structure in the short term can cause the organizational resistance and reduced performance. Thus, we assume that the efficiency score of DMU$_i$ with new input and output values $(\hat{\mathbf{x}}_i,\hat{\mathbf{y}}_i)$ set by (3) and (4) is the same as the efficiency score of DMU$_i$ with the observed input and output values $(\mathbf{x}_i,\mathbf{y}_i)$.

3. Centralized resource allocation model based on efficiency analysis

In this section, we consider a decision-making environment in which a central unit can simultaneously control all of the units. The aim of the decision-maker is to reduce the total input consumption of all of the DMUs in such a way that the efficiency score of all of the DMUs with new input and output targets remains unchanged and the same total outputs are achieved. Let z_i^* be the relative inefficiency value of DMU$_i$ by model (1). The centralized resource-allocation model based on the efficiency analysis is formulated as the following linear program:

$$\begin{aligned} & \text{Min } 1-z \\ \text{s.t. } & \sum_{i=1}^{n} \mathbf{X}\eta_i \le (1-z)\mathbf{X}\mathbf{1} \\ & \sum_{i=1}^{n} \mathbf{Y}\eta_i \ge \mathbf{Y}\mathbf{1} \\ & \mathbf{X}\mu_i \le (1-z_i^*)\mathbf{X}\eta_i \qquad i=1,\cdots,n \\ & \mathbf{Y}\eta_i \le \mathbf{Y}\mu_i \qquad i=1,\cdots,n \\ & \mathbf{1}^T\mu_i = 1 \qquad i=1,\cdots,n \\ & \mathbf{1}^T\eta_i = 1 \qquad i=1,\cdots,n \\ & \eta_i,\mu_i \ge 0 \qquad i=1,\cdots,n \end{aligned} \tag{5}$$

where $\mu_i=[\mu_{i1},\mu_{i2},\cdots,\mu_{in}]^T \in R^n_+$. The first constraints seek to reduce the total input consumption of all the DMUs as much as possible. The second constraints ensure that the same total outputs are achieved. The third and fourth constraints guarantee that the relative inefficiency value of the projected point $(\mathbf{X}\eta_i,\mathbf{Y}\eta_i)$ for DMUi for $i=1,\cdots,n$ after resource allocation must be equal to z_i^*.

Theorem. *In model (3), the relative inefficiency value of the projected point $(\mathbf{X}\eta_i,\mathbf{Y}\eta_i)$ for DMU$_i$ for $i=1,\cdots,n$ after resource allocation must be equal to z_i^*, that is, the efficiency score for each inefficient DMU after resource allocation remains unchanged.*

Proof. Let $1-z^*$ be the optimal value and $\mu_i^*(i=1,2,\cdots n)$ and $\eta_i^*(i=1,2,\cdots n)$ be the optimal solutions to the model (3), respectively.