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Simultaneous measurement of small birefringence magnitude and direction in real time

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ABSTRACT

A method to simultaneously measure the small birefringence magnitude and direction in real time is proposed. The laser light passes through a circular polarizer and the sample successively. Then the beam is split into two sub-beams by a beam splitter. Each *sub beam* is then split and analyzed by a Wollaston prism. Four intensities are simultaneously detected *by four detectors* to resolve the birefringence magnitude and direction in real time. The measurement result is immune to fluctuation of the initial intensity. In experiments, a photoelastic modulator with small peak retardation was measured at different birefringence magnitudes and directions. The maximum deviation of the birefringence magnitude is less than 1.68 nm, and the standard deviation of the birefringence direction is less than 0.47°. The validity of the method is verified.

1. Introduction

Birefringence is the property of anisotropic optical materials. When two orthogonal linear polarized lights pass through the birefringence sample, the difference of two lights' speed will generate phase shift between them. The net phase shift along the path of the two lights is the birefringence magnitude. The polarization direction with the greatest speed of light is the birefringence direction. There are considerable interests in measuring the birefringence of the liquid crystal [1,2], optical glass [3], wave plate [4], optical fiber [5–7] and biological specimen [8–10]. Several techniques for measuring birefringence magnitude and direction have been developed, including intensity determined method [11–13], compensator-based method [14–17], and phase modulated method [18–20]. The intensity determined method needs to rotate the polarizer or the retarder to calculate the birefringence. The compensator-based method generally changes the compensating amount to make the detected signal to zero or a special value. The phase modulated method needs to analyze the modulated signal in frequency domain. The phase modulator includes photoelastic modulator [18], electro-optic modulator [19] and stress-optic modulator [20]. Due to the rotation of the polarization component, the movement of the compensating plate and the sampling of the modulation signals, the above methods are not suitable for real time birefringence measurement. For those optical systems whose birefringence is dynamically changing [21,22], real-time measurement of the birefringence magnitude and direction becomes increasingly important. In this paper, a method to simultaneously measure the birefringence magnitude and direction in real time is proposed.

2. Principle

The schematic diagram of the method is shown in Fig. 1. The optical layout includes a laser, a circular polarizer, the sample to be measured, a beam splitter, two Wollaston prisms and four detectors. For convenience, z-axis of the coordinate system is chosen as the light propagation direction and x-axis is along the vertical direction. The collimating laser beam passes through the circular polarizer and becomes circularly polarized light. The circularly polarized light passes through the sample and then is split into two sub-beams by the beam splitter. The sub-beam transmitted through the beam splitter is then split by Wollaston prism 1. Wollaston prism 1 can be seen as two linear analyzers, whose two transmission axis azimuths are respectively 45° and 135°. Two sub-beams after Wollaston prism 1 are then detected by detector 1a and detector 1b. Similarly, the other sub-beam reflected by the beam splitter with a small angle is splitted and by Wollaston prism 2. Wollaston prism 2 can also be seen as two analyzers, whose two transmission axis azimuths are respectively 0° and 90°. Two sub-beams after Wollaston prism 2 are detected by detector 2a and

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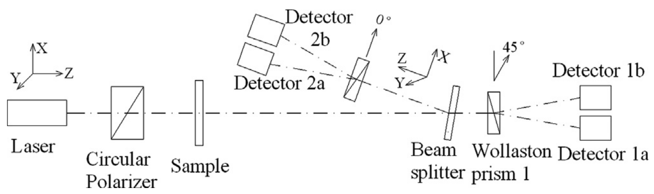

Fig. 1. Schematic diagram for real-time measurement of birefringence magnitude and direction.

detector 2b. By analyzing the four signals of the detectors, the birefringence magnitude and direction of the sample can be obtained in real time.

The laser beam passing through the circular polarizer becomes circularly polarized light. Its Jones vector E_0 can be written as [23]

$$E_0 = \sqrt{\frac{I_0}{2}}\begin{bmatrix}1\\ i\end{bmatrix}, \tag{1}$$

where I_0 is the initial intensity. The Jones matrix of the sample is expressed as [23]

$$J_s = \begin{bmatrix}\cos\frac{\delta}{2} - i\sin\frac{\delta}{2}\cos 2\theta & -i\sin\frac{\delta}{2}\sin 2\theta \\ -i\sin\frac{\delta}{2}\sin 2\theta & \cos\frac{\delta}{2} + i\sin\frac{\delta}{2}\cos 2\theta\end{bmatrix}, \tag{2}$$

where δ is the birefringence magnitude (retardation) and θ is the birefringence direction (fast axis azimuth) of the sample. The Wollaston prisms split the input beam and can be seen as two linear analyzers. The Jones matrix of the Wollaston prisms can be expressed as [23]

$$J_\alpha = \begin{bmatrix}\cos^2\alpha & \frac{1}{2}\sin 2\alpha \\ \frac{1}{2}\sin 2\alpha & \sin^2\alpha\end{bmatrix}, \tag{3}$$

where α is the transmission axis azimuth. Jones vectors E_{1a}, E_{1b}, E_{2a} and E_{2b} of the beams on detector 1a, detector 1b, detector 2a and detector 2b can be given respectively by

$$E_{1a} = K_1 J_\alpha J_s E_0\big|_{\alpha=45}, \tag{4}$$

$$E_{1b} = K_1 J_\alpha J_s E_0\big|_{\alpha=135}, \tag{5}$$

$$E_{2a} = K_2 J_\alpha J_s E_0\big|_{\alpha=0}, \tag{6}$$

$$E_{2b} = K_2 J_\alpha J_s E_0\big|_{\alpha=90} \tag{7}$$

where K_1 is the transmissivity of the beam splitter, K_2 is the reflectivity of the beam splitter. The intensities I_{1a}, I_{1b}, I_{2a} and I_{2b} on detector 1a, detector 1b, detector 2a and detector 2b can be obtained simultaneously by

$$I_{1a} = E_{1a}E_{1a}^* = \frac{1}{2}K_1^2 I_0[1 - \sin(\delta)\cos(2\theta)], \tag{8}$$

$$I_{1b} = E_{1b}E_{1b}^* = \frac{1}{2}K_1^2 I_0[1 + \sin(\delta)\cos(2\theta)], \tag{9}$$

$$I_{2a} = E_{2a}E_{2a}^* = \frac{1}{2}K_2^2 I_0[1 + \sin(\delta)\sin(2\theta)], \tag{10}$$

$$I_{2b} = E_{2b}E_{2b}^* = \frac{1}{2}K_2^2 I_0[1 - \sin(\delta)\sin(2\theta)]. \tag{11}$$

with above Eqs. (8)–(11), we can get

$$V_1 = \frac{I_{1a} - I_{1b}}{I_{1a} + I_{1b}} = \sin(\delta)\cos(2\theta), \tag{12}$$

$$V_2 = \frac{I_{2a} - I_{2b}}{I_{2a} + I_{2b}} = \sin(\delta)\sin(2\theta). \tag{13}$$

Thus the initial intensity I_0, coefficients K_1 and K_2 are removed from the final result. Thus the birefringence magnitude can be acquired with

$$\delta = \arcsin\left(\sqrt{V_1^2 + V_2^2}\right), \tag{14}$$

When the birefringence magnitude is less than 45°, the sine function is closely linear. Its nonlinear measurement error is very small. Thus the measurement range can be defined as 0–$\lambda/8$, i.e. 0–45°. And the birefringence direction of the sample can be calculated theoretically by

$$\theta' = \frac{1}{2}\arcsin\frac{V_2}{\sqrt{V_1^2 + V_2^2}}. \tag{15}$$

or

$$\theta'' = \frac{1}{2}\arccos\frac{V_1}{\sqrt{V_1^2 + V_2^2}}. \tag{16}$$

When the range of θ' and θ'' is respectively –22.5° to 22.5° and 22.5° to 67.5°, they can be considered as closely linear.

The birefringence direction can be expended to the range of 0–180°. Firstly the absolute value of V_1 and V_2 is compared. If $|V_1| > |V_2|$, the arcsine function is used to determine the birefringence direction as follow

$$\theta = \begin{cases}\theta', & |V_1| > |V_2|, \quad V_1 > 0,\ V_2 > 0 \\ \theta' + 90^\circ, & |V_1| > |V_2|, \quad V_1 < 0 \\ \theta' + 180^\circ, & |V_1| > |V_2|, \quad V_1 > 0,\ V_2 < 0\end{cases}, \tag{17}$$

If $|V_1| \le |V_2|$, the arccosine function is used and the birefringence direction is calculated as

$$\theta = \begin{cases}\theta'', & |V_1| \le |V_2|,\ V_2 > 0 \\ \theta'' + 90^\circ, & |V_1| \le |V_2|,\ V_2 < 0\end{cases}. \tag{18}$$

The intensities on detector 1a, detector 1b, detector 2a and detector 2b are detected and processed at the same time, thus the birefringence magnitude and direction can be obtained in real time. Moreover, the measured results are immune to the fluctuation of the initial intensity, which ensures this method suitable for real-time measurement.

3. Experiment and measurement

The experimental setup is shown in Fig. 1. The light source was a He–Ne laser with the wavelength of 632.8 nm. The circular polarizer consisted of a polarizer and a quarter-wave plate. The polarizer was a Glan–Taylor prism whose extinction ratio was better than 100,000:1. The quarter-wave plate was a zero-order quarter-wave plate with the retardation tolerance of less than $\lambda/300$ (λ=632.8 nm). The beam splitter had a splitting ratio of 1:1. The two Wollaston prisms' splitting angles were both 5° and their extinction ratios were better than 100,000:1. The detector consisted of a photo-diode and a pre-amplifier. The pre-amplifier was specifically designed for high frequency optical signals. The detectors were then connected with the data acquisition card (PCI 6133, National Instruments).

The sample was a PEM100 photoelastic modulator produced by HINDS INSTRUSMENT. Its birefringence magnitude was sinusoidally modulated with 40 kHz frequency. The maximum birefringence magnitude (peak retardation) could be changed by the PEM-100 controller. The PEM100 photoelastic modulator is well calibrated before measurement. For example, the modulated birefringence magnitude with the peak retardation of 0.05λ (31.64 nm) is shown in Fig. 2a. The wave form is of perfect sinewave. In fact, only the positive birefringence magnitude is considered [1–10]. According to Eq. (14), the birefringence magnitude is the absolute value of the