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One shot profilometry using a composite fringe pattern

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A method is proposed to solve one of the problems that profilometry encounters when fringe projection techniques are used: the difficulty to discern between surface discontinuities that cause phase shifts greater than 2π. Based on fringe projection of a single composite fringe pattern containing three different frequencies, such problem can be solved. Experimental results are presented using Fourier methods.

1. Introduction

There is a diversity of techniques to get the shape of objects that have surface discontinuities or spatially isolated surfaces through the projection of light; perhaps the simplest is the projection of a single line, with the disadvantage of needing a scanning process [1]. In general, most methods, such as temporal phase unwrapping [2] or multi-frequency fringe projection [3], require the manipulation of various images; a detailed description of them can be found on the review articles of Refs. [4] and [5]. In order to reduce the erroneous measurements caused by object movements during the image grabbing process, or to decrease the acquisition time, it is desirable to get the 3D shape information from a single image. Takeda et al. [6] have proposed a method based on a two-frequency fringe pattern and modifications to the Gushov–Solodkin algorithm. Later, other single-shot methods have been proposed, as the two-frequency fringe pattern of [7], which requires an unwrapping process for both the high and low frequency phases. Also, a single color pattern has been proposed [8], but it requires an additional calibration process to avoid color crosstalk; moreover, the color absorption of some materials restricts its use. Additional references that utilize one image to get the surface profile can be found in the reviewed article of Ref. [9].

Our proposal deals with the projection of a single image (gray levels) to get the shape of objects having discontinuities or being spatially isolated. It is based on the projection of a composite fringe pattern with three frequencies and the calculation of a fringe pattern with an equivalent period of one. The phase of this equivalent pattern gives us the phase without using an unwrapping process. By scaling this phase, we can get the fringe orders of the high frequency components irrespective of phase discontinuities. These fringe orders help us to unwrap the phase and to determine its ambiguities caused by phase jumps greater than 2π. This method resembles the one proposed in Ref. [10], with the difference that they project the single period and the high frequency patterns independently; also, to get its phase values, they use phase shifting techniques, which requires at least three images of each one.

2. Theory

Let us explain our proposal. Using a computer, we generate a composite pattern to be projected onto the object surface given by

$$i_D(x,y) = (G/6)\{3 + \cos(2\pi f x) + \cos(2\pi f y) + \cos[2\pi(f+1)x + 2\pi f y]\}, \quad (1)$$

where f is a carrier frequency, G is a constant that represents the amplitude value introduced to obtain the maximum gray level range (i.e. $G = 255$ for eight bit images), (x,y) are the normalized pixel coordinates, and $i_D(x,y)$ is the image with its gray levels in the range $[0,G]$. It can be noticed that the pattern given by Eq. (1) comprises the sum of three fringe patterns: one with vertical fringes, another with horizontal fringes, and the last one with fringes almost at 45°. If we denote the carrier terms as follows,

$$c_x(x,y) = 2\pi f x; \quad c_y(x,y) = 2\pi f y; \quad c_{xy}(x,y) = 2\pi(f+1)x + 2\pi f y \quad (2)$$

then the following relation holds,

$$c_{xy}(x,y) - c_x(x,y) - c_y(x,y) = 2\pi x, \quad (3)$$

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its cosine is a one period vertical fringe. This is an important relation that we will use later on. To simplify our notation, we will drop off the (x, y) variables, but they will be implicit, although in some formulas, they are included to emphasize their dependence.

The intensity profile that we will obtain after projecting Eq. (1) onto the object's surface will be given by

$$i = a + b[\cos(c_x + \varphi^x) + \cos(c_y + \varphi^y) + \cos(c_{xy} + \varphi^{xy})], \tag{4}$$

where a and b are background and amplitude terms that depend on the object's reflectivity, respectively, and φ^x, φ^y and φ^{xy} are the phase functions related to the surface height $h(x, y)$. Eq. (4) can be rewritten in the following form

$$i = a + d_x \exp(jc_x) + d_y \exp(jc_y) + d_{xy} \exp(jc_{xy}) + d_x^* \exp(-jc_x) + d_y^* \exp(-jc_y) + d_{xy}^* \exp(-jc_{xy}), \tag{5}$$

where $d_x = (1/2)b \exp(j\varphi^x)$, $d_y = (1/2)b \exp(j\varphi^y)$, $d_{xy} = (1/2)b \exp(j\varphi^{xy})$, $j = \sqrt{-1}$ and $*$ means complex conjugated.

The Fourier transform of Eq. (5) can be expressed as:

$$I(u, v) = A(0, 0) + D_x(u-f, v) + D_y(u, v-f) + D_{xy}(u-f-1, v-f) + D_x^*(u+f, v) + D_y^*(u, v+f) + D_{xy}^*(u+f+1, v+f), \tag{6}$$

where (u, v) are the frequency coordinates, and f is the carrier frequency of Eq. (1). Eq. (6) consists of seven spectrums (denoted by capital letters) centered on frequencies (0,0), (f,0), (0,f), ($f+1$,f), ($-f$,0), (0,$-f$) and ($-f-1$,$-f$). Since the distances from center to center of the spectrums depend on frequency f, no overlap among them occurs for f that is big enough. Because each spectrum and its conjugate contain the same phase information (except for their opposite sign), we select only D_x, D_y and D_{xy} to be filtered. The separations of those terms are carried out by a band-pass filter, and then transformed into the space domain by the inverse of the Fourier transform. The respective phases (modulus 2π) are obtained with the arctg of the imaginary part over the real part. This way we end up with the following three phases (modulus 2π):

$$\Phi^x = [c_x + \varphi'^x]_{\text{mod } 2\pi} = \text{arctg}\{\text{Im}[D_x(u-f, v)]/\text{Re}[D_x(u-f, v)]\}, \tag{7a}$$

$$\Phi^y = [c_y + \varphi'^y]_{\text{mod } 2\pi} = \text{arctg}\{\text{Im}[D_y(u, v-f)]/\text{Re}[D_y(u, v-f)]\}, \tag{7b}$$

$$\Phi^{xy} = [c_{xy} + \varphi^{xy}]_{\text{mod } 2\pi} = \text{arctg}\{\text{Im}[D_{xy}(u-f-1, v-f)]/\text{Re}[D_x(u-f-1, v-f)]\}, \tag{7c}$$

hence, the wrapped difference of Φ^{xy}, Φ^x and Φ^y is given by

$$\Phi^w = \text{arctg}\left[\frac{\sin(\Phi^{xy} - \Phi^x - \Phi^y)}{\cos(\Phi^{xy} - \Phi^x - \Phi^y)}\right], \tag{8}$$

which, by Eq. (3), consists of only one period, and because of the arctg function, is within the range 0 to 2π. Following with our procedure, it is important to get Φ, the unwrapped function of the low frequency wrapped function Φ^w. If the object height under test is low enough, we may have $\Phi^w = \Phi$ directly, without any additional unwrapping procedure; otherwise, it will be necessary to use an unwrapping technique. In any case the following relation is satisfied

$$\Phi = (c_{xy} - c_x - c_y) + (\varphi^{xy} - \varphi^x - \varphi^y) \tag{9}$$

Using Eq. (3) in Eq. (9) we obtain

$$\Phi(x, y) = 2\pi x + \varphi^{Eq}(x, y), \tag{10}$$

where $\varphi^{Eq} = \varphi^{xy} - \varphi^x - \varphi^y$ represents the equivalent phase of the phase differences. Then, what we have obtained is the phase $\Phi(x, y)$ of the projection of a vertical fringe pattern with one period. However, as it is known [11], the phase and the height surface can be approximated by

$$\varphi(x, y) = 2\pi f h(x, y)/D, \tag{11}$$

where D is a constant that depends on the camera and projector positions, $h(x, y)$ is the surface height, and f is the frequency of the carrier fringes (in Eq. (10), $f = 1$). We considered $D = l/d$, where d is the separation between the camera and the projector and l is the distance from the projector to the surface of the reference plane.

Since the surface height calculated from the known phase function depends on the frequency of the projected fringes, $h(x, y) = D\varphi(x, y)/2\pi f$, it is desired to project fringes with frequencies as high as possible in order to get also higher resolution. In our case, the phase Φ^x calculated with Eq. (7a) was obtained with a carrier frequency f; then, in principle, it is f-times more sensitive than the phase Φ obtained with Eq. (10), with the drawback that Φ^x is a wrapped phase function. Then, we will describe a procedure to get Φ^x provided Φ, or more precisely, to unwrap Φ^x given Φ.

From Eqs. (2), (4) and (10), we can see that multiplying $\Phi(x, y)$ by f gives us an approximate value of the unwrapped phase $\Phi^x(x, y)$. On the other hand, the fringe order number $N(x, y)$ relates the wrapped $\Phi^x(x, y)$ and unwrapped $\Phi(x, y)$ phase functions with the equation

$$N(x, y) = [f \;\; \Phi(x, y) - \Phi^x(x, y)]/2\pi; \tag{12}$$

then, we can unwrap Φ^x using the equation

$$\Phi_u^x(x, y) = \Phi^x(x, y) - 2\pi N(x, y), \tag{13}$$

where $\Phi_u^x(x, y)$ is the unwrapped phase of $\Phi^x(x, y)$.

We can also obtain the fringe orders of $\Phi^x(x, y)$ in the following way,

$$N_0(x, y) = [f 2\pi x - \Phi^x(x, y)]/2\pi, \tag{14}$$

to get another unwrapped estimation of $\Phi^x(x, y)$ given by

$$\Phi_{0u}^x(x, y) = \Phi^x(x, y) - 2\pi N_0(x, y), \tag{15}$$

with the drawback that $\Phi_{0u}^x(x, y)$ is not sensitive to phase jumps greater than 2π, as $\Phi_u^x(x, y)$ is. $N_0(x, y)$ and $N(x, y)$ are important functions that we will use in Section 3 to discard undesired error introduced by our method.

An important aspect is the maximum dynamic range that we can work with. From Ref. [11], we know that for a traditional one direction sinusoidal projected fringe of frequency f and maximum background frequency f_b, the following condition is obtained (only two spectrums interact):

$$\left|\frac{\partial\varphi(x, y)}{\partial x}\right| < 2\pi(f - f_b), \tag{16}$$

However, in our case, we will have the interactions among the four spectrums $A(0, 0)$, $D_x(u-f, v)$, $D_y(u, v-f)$ and $D_{xy}(u-f-m, v-f)$ (Eq. (6)). Since the separation among the centers of those spectrums is greater or equal to f, the cutoff frequency radius of each spectrum must be greater than or equal to $f/2$. Then, from the definition of local frequencies along x and y directions, $2\pi f_x(x, y) = \partial\varphi(x, y)/\partial x$ and $2\pi f_y(x, y) = \partial\varphi(x, y)/\partial y$, respectively, the new conditions for our algorithm are

$$\left|\frac{\partial\varphi(x, y)}{\partial x}\right| < 2\pi(f - f_b), \tag{17a}$$

$$\left|\frac{\partial\varphi(x, y)}{\partial y}\right| < 2\pi\left(\frac{f}{2}\right), \tag{17b}$$

Given that in general $f_b < f/2$ we can simplify the conditions to be

$$|\partial\varphi(x, y)/\partial x| < 2\pi(f/2) \text{ and } |\partial\varphi(x, y)/\partial y| < 2\pi(f/2) \tag{17c}$$

The maximum slope (steep of the surface) measurable without ambiguity can be estimated from Eqs. (11) and (17a)–(17c). Deriving Eq. (11) and substituting in the previous two equations