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2-D Continuous Wavelet Transform for ESPI phase-maps denoising

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ABSTRACT

In this work we introduce a 2-D Continuous Wavelet Transform (2-D CWT) method for denoising ESPI phase-maps. Multiresolution analysis with 2-D wavelets can provide high directional sensitivity and high anisotropy which are proper characteristics for this task. In particular, the 2-D CWT method using Gabor atoms (Gabor mother wavelets) which can naturally model phase fringes, has a good performance against noise and can preserve phase fringes. We describe the theoretical basis of the proposed technique and show some experimental results with real and simulated ESPI phase-maps. As can be verified the proposal is robust and effective.

1. Introduction

Optical methods are widely used for nondestructive testing and metrology. In particular, Electronic Speckle Pattern Interferometry (ESPI) is used to measure stress, deformation, vibration, etc. Important steps in ESPI techniques are the demodulation of fringe patterns and phase unwrapping. Phase unwrapping is a technique used to remove the 2π phase that jumps in the wrapped phase-map. Recovering the continuous phase-map is usually a big challenge [1], however, ESPI phase-maps are often very noisy. This characteristic hinders an easy and reliable phase unwrapping, therefore a proper filtering is often required.

Recently, several phase-map denoising techniques have been proposed. In most of these works it is remarked that the importance of filtering along the fringe orientation in order to preserve the fringes. Tang et al. [2–4] have reported some interesting techniques based on variational methods solving oriented partial differential equations (OPDE). In our previous works we also proposed a filtering technique using the regularization theory: the Regularized Quadratic Cost Function (RQCF) [5,6]. However, a drawback of all mentioned techniques is that they require the previous estimation of the so-called fringe orientation which, as it uses the computation of the image gradient, could be an inaccurate procedure in the presence of noise and low-modulation of fringes. A typical characteristic of the resulting filtered phase image is the presence of structures due to the changes in the orientation angles, which is also a drawback. Others techniques such as the Localized Fourier Transform (LFT) filter for noise removal in ESPI phase-maps [7] and the Windowed Fourier Transform (WFT) [8] are techniques that work effectively without a previous estimation of the fringe orientation. However, in the case of the WFT technique eight parameters have to be adjusted depending on the phase image and it requires a long processing time.

The 1-D Discrete Wavelet Transform (1-D DWT) is a tool that provides local, sparse and decorrelated multiresolution analysis of images, these properties being very much exploited for denoising. However, the 1-D wavelet methods present some strong limitations that reduce their effectiveness in two dimensions. Although 1-D wavelets have impacted in image processing they do not efficiently represent elements with high anisotropy, as in the case of wrapped phase-maps. The reason is that 1-D wavelets are non-geometrical and do not properly model the regularity of such structures. Therefore, the most serious disadvantage of the 1-D DWT in phase-map denoising is its poor directionality. For instance, in the work presented by Kauffmann et al. [9], the authors present a method to reduce the speckles in TV holographic fringes using Daubechies wavelets with thresholding. Such wavelets, however, are not very effective at image edges. Also, Shakher et al. [10] proposed a method using Symmetric Daubechies wavelets, however, authors do not present explicit formulas.

In the last decade, 2-D wavelets [11] [12] have been used as a proper alternative to the weakness of the 1-D DWT to sparsely representing elements with high anisotropy. The 2-D CWT has already been used for interferogram demodulation [12]. In particular, the 2-D CWT using Gabor atoms (Gabor atoms are composed of complex periodic functions modulated by Gaussian functions) can naturally model phase fringes. For this reason we propose a

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2-D CWT method for phase-map denoising. As will be shown, this proposal has a high performance against noise similar to the LFT and WFT methods. Additionally, we provide the MATLAB function code in [13].

This paper is organized in the following way. In Section 2 we describe the mathematical fundamentals of the proposed method. In Section 3 we describe a practical implementation of the 2-D CWT denoising method. In Section 4 we present some experimental results. In Section 5 we present a discussion of the experimental results. Finally in Section 6 we present the conclusions.

2. The 2-D Continuous Wavelet Transform

The wavelet transform has become a standard tool in signal and image processing, and it has found applications in almost all fields of physics, engineering and applied mathematics. The 2-D CWT provides localized spectral information of the analyzed dataset and it covers the domain of the analyzed data with a continuous analysis from which detail, shift-invariant spectral information of different positions and orientations can be obtained. The 2-D CWT is used for analysis and features detection in images, with special emphasis on the detection of singularities (contours, sharp transitions, etc.).

Mathematically, images can be represented as two dimensional arrays in which each element represents the intensity. For our purposes, a wrapped phase-map ϕ may be represented as a complex image

$$I(x,y) = e^{i\phi(x,y)} = \cos\phi(x,y) + i\sin\phi(x,y). \quad (1)$$

Then, its 2-D continuous wavelet decomposition can be defined as

$$CWT(\xi,\eta,\theta,a) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y)\psi^{*}_{\xi,\eta,\theta,a}(x,y)\,dx\,dy, \quad (2)$$

where $\psi_{\xi,\eta,\theta,a}(x,y)$ represents the family of wavelets, (x,y) are the image coordinates, and $\theta\in(-\pi,\pi]$ the orientation angle. The family of wavelets ψ are shifted by ξ and η, oriented by the angle θ, and scaled by factor a. The $*$ symbol indicates the complex conjugated.

The mother Gabor wavelet in 2-D can be expressed as

$$\psi_{\xi,\eta,\theta,a}(x,y) = \exp\left[-\frac{\pi[(x-\xi)^2+(y-\eta)^2]}{a}\right] \times\exp\left[i2\pi\frac{f}{a}[(x-\xi)\cos\theta+(y-\eta)\sin\theta]\right], \quad (3)$$

where f is the frequency (Fig. 1).

Substituting (1) and (3) in (2) we obtain

$$CWT(\xi,\eta,\theta,a) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp[i\phi(x,y)] \times\exp\left[-\frac{\pi[(x-\xi)^2+(y-\eta)^2]}{a}\right] \times\exp\left[-i2\pi\frac{f}{a}[(x-\xi)\cos\theta+(y-\eta)\sin\theta]\right]dx\,dy. \quad (4)$$

To simplify Eq. (4), let $x'=x-\xi$, $y'=y-\eta$ and $(\chi,\gamma)=(f/a\cos\theta, f/a\sin\theta)$, where (χ,γ) represents a vector of frequencies in the direction of θ. Using Taylor's expansion we know that

$$\phi(x'+\xi,y'+\eta)\approx\phi(\xi,\eta)+\left[\frac{\partial\phi}{\partial\xi}x'+\frac{\partial\phi}{\partial\eta}y'\right]. \quad (5)$$

Then, we can approximate Eq. (4) as follows:

$$CWT(\xi,\eta,\theta,a)\approx\exp[i\phi(\xi,\eta)]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{i\left[\frac{\partial\phi}{\partial\xi}x'+\frac{\partial\phi}{\partial\eta}y'\right]\right\}\times\exp\left[-\pi\frac{(x'^2+y'^2)}{a}\right] \times\exp[-i2\pi(\chi x',\gamma y')]\,dx'\,dy'. \quad (6)$$

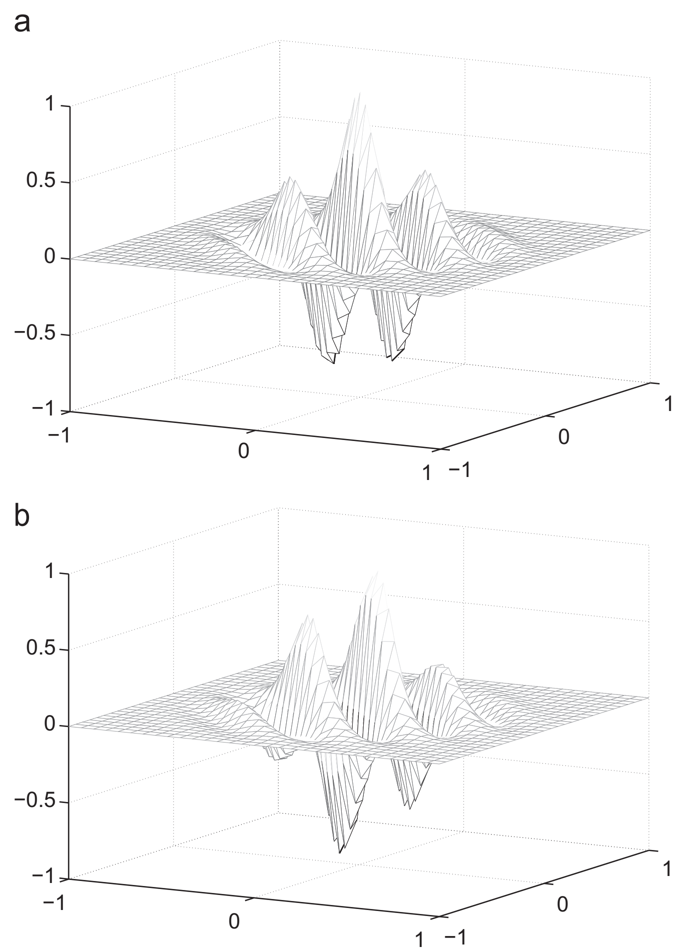

Fig. 1. Example of a 2-D Gabor wavelet at $\theta=\pi/4$. (a) Real part, (b) imaginary part.

The integral represents the Fourier transform of a 2-D complex periodic function with frequency $\nabla\phi(\xi,\eta)/2\pi$, modulated by a scaled Gaussian function. Finally, after applying proper Fourier theorems we find that

$$CWT(\xi,\eta,\theta,a)\approx\exp[i\phi(\xi,\eta)] \times\exp\left\{-a\pi\left[\left(\chi-\frac{1}{2\pi}\frac{\partial\phi}{\partial\xi}\right)^2+\left(\gamma-\frac{1}{2\pi}\frac{\partial\phi}{\partial\eta}\right)^2\right]\right\}. \quad (7)$$

From the mathematical point of view, the 2-D CWT using Gabor atoms provides a local spectral energy density concentrated around a given position in the frequency domain, which is adequate to model the phase image. It is remarkable that the 2-D CWT represents a very detailed frequency and oriented decomposition of $I(x,y)$. Fig. 2 shows that the transformation is performed along different directions and frequencies. The frequency value at the center of circle is equal to zero. The orientation θ is used to describe angles in the range of $(-\pi,\pi]$.

2.1. Denoising with the 2-D CWT

Using 2-D CWT spurious information or noise may be efficiently removed in the following way: Consider that for every (ξ,η) of the CWT we find the maximum magnitude of the 2-D (θ,a) coefficients map, which represents the local phase-fringe-orientation and frequency. We call this maximum (θ_r,a_r) and the 2-D (ξ,η,θ_r,a_r) map is called the *ridge* of the 2-D CWT. By extracting the ridge, all the coefficients contributed by the noise are removed.