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Measurement of dynamic properties of small volumes of fluid using MEMS

David Cheneler [a,*], Michael C.L. Ward [a], Michael J. Adams [b], Zhibing Zhang [b]

[a] *Department of Mechanical Engineering, The University of Birmingham, Edgbaston, Birmingham B15 2TT, UK*

[b] *Department of Chemical Engineering, The University of Birmingham, Edgbaston, Birmingham B15 2TT, UK*

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Abstract

A model based on the response of a micro-rheometer which permits the measurement of the linear viscoelastic properties of small volumes of a fluid is described. The configuration involves a liquid being contained within a capillary bridge between two flat smooth parallel platens that are actuated sinusoidally using a compliant MEMS device. Approximate closed-form equations are derived to analyse the data taking account of both the capillary forces and those arising from viscoelastic flow. The approximate theory is compared to a full numerical simulation of the response of the MEMS rheometer and the validity is discussed.

Keywords: Squeeze flow; Capillary bridges; MEMS; Viscoelastic properties; Complex fluid

1. Introduction

There is often a need to measure the properties of liquid. Occasionally the available volume of the liquid of interest may be sufficiently small as to render conventional methods of rheometry such as cone and plate rheometry [1], stormer viscometry [2] or falling ball viscometry [3] inappropriate. Consequently, there is a growing interest in the use of MEMS devices to measure the required properties, especially with an aim of encouraging high throughput. These devices include pressure sensors [4], optical tweezers [5] and micro-particle image velocimetry [6] amongst others [7–11]. The current paper examines the potential of employing MEMS for making a rheometer based on squeeze flow, which is a convenient configuration for this technology. In particular, the design and analysis of a device based on sinusoidal oscillation for viscoelastic fluids in the linear strain region will be described. As will be seen, the linearisation of the model requires the use of small amplitudes, which precludes the use of traditional rheometers making this model particularly suitable for use with micro system technology.

The analysis of squeeze flow rheometry is an area of continuing development in order to apply the method to fluids with more complex constitutive behaviour [12–18] than those that exhibit simple Newtonian flow. Generally steady rather than oscillatory flow has been considered although there are notable exceptions [14,16,19]. If the platens are not fully immersed so that the liquid is contained as a discrete bridge, it is necessary to consider the influence of the capillary forces that may be important for small viscosities and platen displacement velocities. Also if the micro-rheometer is based on a compliant oscillating device, the mechanical properties of the device need to be taken into account. These aspects have been neglected in previous work and will be considered here.

2. The micro-rheometer

There are many possible different configurations that could be employed for a micro-rheometer [6–12]. It is not the purpose of this paper to suggest another alternative but rather to

* Corresponding author.
E-mail address: dxc189@bham.ac.uk (D. Cheneler).

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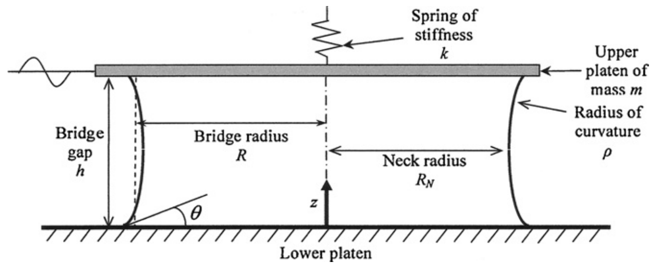

Fig. 1. Schematic of liquid bridge within the MEMS device.

consider the relevance of squeeze flow theory to micro-systems technology. Consequently, here a generic device will be considered that comprises a fixed lower platen and a parallel rigid upper platen with a known mass being supported by an elastic beam or spring of known stiffness (see Fig. 1). It will be assumed that the upper platen is electrostatically actuated with a sinusoidal force of known frequency and magnitude.

The device will also have set operating parameters. It will comprise of electrostatic plates of approximate area 80 mm^2 and gap 2 μm. It is assumed that the oscillation amplitude is in the range 25–250 nm and this can be measured with a resolution of 1 in 100. It is also assumed that the phase difference can be measured between 0 and $\pi/2$ rad with an accuracy of 1°; the alternating voltage varies between 0.001 and 10 V with a 1 in 100 resolution and the frequency varies between 0.1 and 100 rad/s with a 1 in 100 resolution. This specification will allow an estimate to be made of the range of liquid properties that could be measured and also the associated errors.

3. Theory

It is assumed that the liquid is accommodated as a pendular bridge between a fixed lower platen and a sinusoidally modulated upper platen (see Fig. 1). In a method similar to that employed by Bell et al. [14], the dynamic properties of the liquid can be calculated by measuring the amplitude of the oscillation of the upper platen and the phase difference between the force and displacement of this platen. The geometry of the liquid bridge as shown schematically in Fig. 1 is dependent on the surface tension, the contact angle and the volume of the liquid; it is assumed that the diameters of both platens are greater than the contact diameter of the liquid bridge. The meniscus curvature, ρ, is a function of the gap between the platens since the hydrostatic pressure difference, Δp, must be constant as defined by the Laplace–Young equation (see Eq. (3)).

Viscoelastic and capillary forces act between the platens. Viscoelastic forces act only when the bridge is in motion as in the case of sinusoidal squeeze flow. Generally if the radius of the liquid bridge is at least 10 times greater than the gap, the lubrication approximation can be applied [14]. This requires that the liquid flows predominantly parallel to the surface of the platens with any elongational or transient flows being neglected. This corresponds to a radial pressure driven flow, which is analogous to Poiseuille flow in a tube such that only a shear velocity field exists. For most practical cases fluid inertial effects may be ignored as generally the frequencies of interest are low (<∼250 Hz) or the viscosities are high [15]. The oscillatory force (F_v) arising from the viscoelastic response is given as [14]:

$$F_v = \frac{3\pi\eta^* R^4}{2h^3} i\omega\, e^{i\omega t} \tag{1}$$

where

$$\eta^* = \eta' - \frac{iG'}{\omega} \tag{2}$$

Here η^* and η' are the complex and dynamic viscosities of the fluid, ω is the angular frequency of oscillation G' is the storage modulus of the liquid, h is the current gap between the platens and t is the time. Due to the curvature of the bridge profile, the radius of the bridge varies along the z-axis. However, it is assumed in the derivation of Eq. (1) that the bridge is cylindrical [14]. Here, to simplify the calculations for the viscoelastic force, the bridge radius, R, will be taken as the average value which is equivalent to the radius of a cylinder of the same volume.

The viscoelastic force acts to resist motion during either the approach or separation of the platens. Strictly the above equations apply to a fixed value of the bridge radius. In the current scheme, this radius will vary sinusoidally. However, the assumption will be made that the amplitude of oscillation will be small compared to the gap so that the error involved will be small.

The total capillary force, F_C, is the sum of that due to surface tension and that associated with the pressure difference arising from the curvature across the liquid/vapour interface as given in Eq. (3). It cannot be calculated analytically except in a few special cases (for example cylindrical geometries and flat planes [21]) and may be written as [20]:

$$F_C = 2\pi R_N \gamma_{lv} - \pi R_N^2 \gamma_{lv} \left(\frac{1}{R_N} - \frac{1}{\rho} \right) \tag{3}$$

where γ_{lv} is the surface tension of the liquid, R_N is the neck radius and ρ is the other principal radius of curvature of the liquid bridge.

It is assumed that to a close approximation, the total force exerted by the liquid may be given by the sum of the viscoelastic and capillary forces. The response of the MEMS device is the sum of the fluid forces (Eqs. (1) and (3)) and that arising from the compliance of the device. Thus the complete response is as follows:

$$m\frac{d^2 z}{dt^2} + \frac{3\pi\eta^* R^4}{2h^3}\frac{dz}{dt} + kz + 2\pi R_N \gamma_{lv} - \pi R_N^2 \gamma_{lv} \left(\frac{1}{R_N} - \frac{1}{\rho} \right) = F_0 \sin \omega t \tag{4}$$

where z is the axial coordinate with the origin at the surface of the lower platen (see Fig. 1), F_0 is the driving force, k is the stiffness of the MEMS device and m is the mass of the upper platen. Since the current value of ρ has to be obtained numerically [21], this equation cannot be solved analytically. Even if this were not the case, the radius, height and the curvature of the bridge are non-linear functions of time, which precludes an analytical solution