Optics and Lasers in Engineering 50 (2012) 1316–1320

Contents lists available at SciVerse ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Impact of linear absorption on self-focusing of Gaussian laser beam in collisional plasma

S.T. Navare [a], M.V. Takale [b], S.D. Patil [c,*], V.J. Fulari [a], M.B. Dongare [a]

[a] Department of Physics, Shivaji University, Kolhapur 416004, Maharashtra, India
[b] Department of Physics, Doodhsakhar Mahavidyalaya, Bidri 416208, Maharashtra, India
[c] Department of Physics, Devchand College, Arjunnagar 591269, Maharashtra, India

ARTICLE INFO

Article history:
Received 20 August 2011
Received in revised form
31 January 2012
Accepted 27 February 2012
Available online 4 April 2012

Keywords:
Gaussian beam
Collisional plasma
Self-focusing

ABSTRACT

The authors have investigated impact of linear absorption on self-focusing of Gaussian laser beam in collisional plasma. The nonlinearity in dielectric constant considered herein is mainly due to the elastic electron–ion collisions. A second order differential equation of dimensionless beam width parameter has been derived and solved numerically. It is observed that absorption plays a vital role in self-focusing of laser beams and weakens the stationary oscillatory character of beam-width parameter with distance of propagation. We have also considered the effect of intensity, relative density and parameter characterizing nature of collisions on propagation characteristics.

1. Introduction

Technological development in the field of laser physics has ushered a new era where highly intense lasers are available. This has opened a new vista of novel applications not only in other fields but also in plasmas such as laser-electron acceleration [1–3], inertial confinement fusion [4–6] and ionospheric modification [7–10]. To make these applications feasible, it is desirable that the laser beam should propagate several Rayleigh lengths (R_d). But in vacuum, the laser beam propagation is limited by diffraction characteristic distance $R_d \sim kr_0^2$, where k is the wave-number and r_0 is the laser spot-size in vacuum; hence self-focusing of laser beams occupies a unique place in laser–plasma interaction. Experiments and simulations relevant to self-focusing have proved that an intense laser beam propagates a long distance with self-focusing offsetting the diffraction divergence. The non-uniform distribution of laser beams along the wave front causes the inhomogeneity of refractive index (effective dielectric constant) of medium on account of inherent nonlinearities, which is responsible for the self-focusing of laser beams [11–13]. In plasmas, the ponderomotive force acting on electrons due to the gradient of electric field intensity would redistribute the electrons along the wavefront, which leads to the ponderomotive non-linearity [13]. The relativistic nonlinearity [14] is attributed to the dependence of electron mass on the quiver velocity of the electrons in the laser field. In addition, the enhancement of electron temperature by electric field and consequent redistribution of the electrons induced by Ohmic heating and collisional energy loss would modify the radial variations of the effective dielectric constant; this phenomenon is commonly known as collisional nonlinearity [13]. It has been said [15] that the change in the refractive index of the medium induced by a Gaussian beam due to the ponderomotive force is about δ times that induced by the collisional phenomena and hence much lower in magnitude with relativistic effects being even weaker; here δ is the fraction of excess energy lost by electron in collision with heavy particles. However, when this effect is significant, the absorption of the beam is important, which accounts for the little significance of this mechanism in practical situations.

Theoretical and experimental researchers have conducted many investigations on the interaction between lasers and collisional plasmas. In theoretical studies of collisional plasmas, laser–plasma interactions, fast-electron generation and transport in fast ignition [16], superthermal electron generation [17] and collisional absorption [18] have been investigated. In addition, simulation results of fast ignition with ultrahigh intensity laser [19], stationary laser beam filaments [20], transverse electron susceptibility and the electromagnetic wave absorption [21] have been obtained. Ghanshyam and Tripathi [22] invoked the stimulated Raman scattering instability of laser beam propagating through a collisional plasma in a self-focused filament. Ma [23] investigated electromagnetic wave propagation in highly collisional plasma. Amrita and Sharma [24] analyzed the thermal self-focusing of a laser in collisonal plasma. Sodha et al. [25] investigated focusing

* Corresponding author. Tel.: +91 8338 220112.
E-mail address: sdpatil_phy@rediffmail.com (S.D. Patil).

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doi:10.1016/j.optlaseng.2012.02.015

of electromagnetic beams in collisional plasmas, with finite thermal conduction. Sharma et al. [26] employed an analytical model to describe the spatiotemporal evolution of a laser pulse propagating through a plasma and studied electromagnetic beam profile dynamics in collisional plasma. The filamentation instability in a collisional magnetoplasma with thermal conduction has been reported by Sodha and Faisal [27]. The effect of electron temperature on laser beam propagation in underdense collisional plasma has been studied by Xia et al. [28]. In experimental studies of collisional plasmas, the dynamics of a dense laboratory plasma jet [29] and the stimulated Brillouin and Raman scattering from a randomized laser beam [30] have been reported. In particular, owing to their prospects for wide applicability and their effects on other nonlinear processes, a large number of investigations focus on beam self-focusing in collisional plasma.

In the present paper, attention is being paid to address a theoretical study of the self-focusing of Gaussian laser beam in underdense collisional plasma with arbitrary nonlinearity. In addition to adopting absorption parameter, our study also incorporates the effect of intensity parameter, relative density parameter and parameter characterizing nature of collisions on the propagating variations of beamwidth parameter in collisional plasma. In Section 2, we have set up and derived a differential equation for beam width parameter f through Wentzel–Kramers–Brillouin (WKB) and paraxial approximations using the parabolic equation approach. The results are presented graphically in Section 3 and discussed. Finally, a brief conclusion is added in Section 4.

2. Analysis

Consider the propagation of an electromagnetic wave of angular frequency ω in a homogeneous plasma along the z axis. The initial intensity distribution of the beam is assumed to be Gaussian and is given by

$$EE^{*}=E_0^2\exp\left(-\frac{r^2}{r_0^2}\right), \tag{1}$$

where r is the radial coordinate of cylindrical coordinate system, r_0 is the initial beam width and E is the electric vector satisfying the wave equation

$$\nabla^2 E+\frac{\omega^2}{c^2}\varepsilon E=0, \tag{2}$$

which may be directly derived from Maxwell's equations by neglecting the term $\nabla(\nabla\cdot E)$. For a transverse field, $\nabla\cdot E=k\cdot E=0$, k being the propagation wave vector. Even if E has a longitudinal component the term $\nabla(\nabla\cdot E)$ can be neglected provided

$$\frac{c^2}{\omega^2}\left|\frac{1}{\varepsilon}\nabla^2\ln\varepsilon\right|\ll 1. \tag{3}$$

The condition (3) is observed to be true in almost all the cases of interest in electromagnetic wave propagation in plasmas.

The effective dielectric constant of homogeneous gaseous plasma is significantly modified when a laser beam passes through it and can, in general, be expressed as

$$\varepsilon=\varepsilon_0+\Phi(EE^{*})-i\varepsilon_i, \tag{4}$$

where $\varepsilon_0=1-\Omega^2$ and Φ are the linear and nonlinear parts of the dielectric constant respectively, $\varepsilon_i=\Omega^2(\nu/\omega)$ takes care of absorption and $\Omega=\omega_p/\omega$, with ω_p as the plasma frequency, given by $\omega_p^2=4\pi n_e e^2/m$, where e and m are the charge and rest mass of the electron respectively, and n_e is the density of the plasma electrons in the absence of the beam. We limit ourselves to the case where ε_i is field independent i.e., absorption is linear and $\varepsilon_i\ll\varepsilon_0$ which implies that absorption is weak. The form of the function Φ is different in different physical situations, but the dependence on EE^{*} is a common feature. With increasing beam power the dielectric constant tends to reach its saturation value. The nonlinear and saturating character of the dielectric constant leads to some interesting features in the propagation of the laser beam and has received considerable attention.

The nonlinear dielectric constant for collisional plasma can be expressed as [13]

$$\Phi(EE^{\star})=\Omega^2\left[1-\left(1+\frac{\alpha}{2}EE^{*}\right)^{(s/2)-1}\right], \tag{5}$$

with $\alpha=e^2M/6m^2\omega^2k_BT_0$, where M is the mass of scatterer in the plasma, k_B is Boltzmann's constant, T_0 is the equilibrium plasma temperature and s is a parameter characterizing the nature of collisions. In the case of collisions of the electrons with neutral particles s may be taken as 1 while for collisions of the electrons with ions, it is -3. In this paper we employ the latter case that takes place predominantly in upper ionosphere. The analysis herein is in any case based only on the form of Eq. (5) and not the detailed physics behind it.

Now, introducing $E=A(r,z)\exp(-ikz)$, where $A(r,z)$ is a complex function of its argument, the behavior of the complex amplitude $A(r,z)$ is described by the parabolic equation obtained from the wave equation (2) in the WKB approximation assuming that the variations in the z direction are slower than those in radial direction:

$$-2ik\frac{\partial A}{\partial z}+\nabla_{\perp}^2A+\frac{\omega^2}{c^2}\left[\Phi(EE^{*})-i\varepsilon_i\right]A=0. \tag{6}$$

To solve Eq. (6), we express A as

$$A=A_0(r,z)\exp(-ikS), \tag{7}$$

where A_0 and S are real functions of r and z (S being the eikonal of the beam). Substituting Eq. (7) for A in Eq. (6) and separating the real and imaginary parts, one can obtain

$$2\left(\frac{\partial S}{\partial z}\right)+\left(\frac{\partial S}{\partial r}\right)^2=\frac{1}{k^2A_0}\nabla_{\perp}^2A_0+\frac{\omega_p^2}{\omega^2\varepsilon_0}\left[1-\left(1+\frac{\alpha}{2}EE^{\star}\right)^{(s/2)-1}\right] \tag{8}$$

and

$$\frac{\partial A_0^2}{\partial z}+\left(\frac{\partial S}{\partial r}\right)\frac{\partial A_0^2}{\partial r}+A_0^2\left(\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}\right)-k\frac{\varepsilon_i}{\varepsilon_0}A_0^2=0. \tag{9}$$

Following Akhmanov et al. [11] and Sodha et al. [12,13], the solution for E can be written as $E=A_0(r,z)\exp[-ik(S+z)]$, with

$$A_0^2=\frac{E_0^2}{f^2}\exp\left[-\frac{r^2}{r_0^2f^2}\right]\exp(-2k_iz), \tag{10}$$

$$S=\frac{r^2}{2}\beta(z)+\phi(z), \tag{11}$$

where k_i is the absorption coefficient given by $k_i=k\varepsilon_i/2\varepsilon_0$ with $k=\omega\varepsilon_0^{1/2}/c$ and $\beta(z)=(1/f)(df/dz)$. The parameter β^{-1} may be interpreted as the radius of curvature of the beam and f is the dimensionless beam-width parameter described by the differential equation

$$\frac{d^2f}{d\eta^2}=\frac{1}{f^3}-\frac{\alpha E_0{}^2R_d^2}{2f^3\varepsilon_0r_0^2}\frac{\omega_p^2}{\omega^2}\left(1-\frac{s}{2}\right)\exp(-2k_i'\eta)\left(1+\frac{\alpha E_0{}^2}{2f^2}\right)^{(s/2)-2} \tag{12}$$

where $\eta=z/R_d$ is the dimensionless distance of propagation, $k_i'=k_iR_d$ is the normalized absorption coefficient and $R_d=kr_0^2$ is the Rayleigh length. The absorption can be neglected in the analysis when $\int_0^z(\omega/c)2k_i'dz\ll 1$, this is of course consistent with $\nu\ll\omega$ [31]. Eq. (12) can be solved numerically with appropriate