Optics and Lasers in Engineering 49 (2011) 71–81

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

A space–time approach in digital image correlation: Movie-DIC

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ARTICLE INFO

Article history:
Received 14 May 2010
Received in revised form
19 August 2010
Accepted 24 August 2010
Available online 9 September 2010

Keywords:
Digital image correlation
Displacement field
Strain field
Strain rate field
Velocity field

ABSTRACT

A new method is proposed to estimate arbitrary velocity fields from a time series of images acquired by a single camera. This approach, here focused on a single spatial plus a time dimension, is specialized to the decomposition of the velocity field over rectangular shaped (finite-element) bilinear shape functions. It is therefore assumed that the velocity field is essentially aligned along one direction. The use of a time sequence over which the velocity is assumed to have a smooth temporal change allows one to use elements whose spatial extension is much smaller than in traditional digital image correlation based on successive image pairs. This method is first qualified by using synthetic numerical test cases, and then applied to a dynamic tensile test performed on a tantalum specimen. Improvements with respect to classical digital image correlation techniques are observed in terms of spatial resolution.

1. Introduction

In solid mechanics, digital imaging is used to detect and measure the motion and deformation of objects. From these observations follow various evaluation procedures of mechanical parameters [1]. To achieve this goal, different optical techniques are used [2]. Among them, digital image correlation (DIC) is appealing thanks to its versatility in terms of scales ranging from nanoscopic [3,4] to macroscopic [5,6] observations with essentially the same type of algorithms.

DIC always involves a compromise between spatial resolution and uncertainty [7,8]. As the technique exploits the comparison of zones of interest, or elements between a deformed and a reference image, the information is carried by the pixels contained in those regions. A key characteristic is thus, β, the number of pixels per kinematic degree of freedom. On the one hand, low uncertainties call for a large β (i.e., large elements), but the description of the displacement will be coarse, and hence may be unsuited to capture rapidly varying displacement fields. The resulting systematic error may be prohibitive for a specific application. On the other hand, small elements may be more flexible to account for a complex displacement field, but as the information content, or β, is small, large uncertainties will result. This trade-off has to be solved for every application, depending on the "complexity" of the expected displacement field. However, one may have access to a large number of pictures thanks to camcorders or high-speed cameras. Traditionally, 2D-DIC operates on image pairs [9–11], and hence a long temporal series is of little use. On the contrary, if the overall displacement over the entire time sequence is large, one may have to break the analysis into time intervals that are finally "chained" to obtain the entire displacement field. When updating the reference picture [12], this procedure involves cumulative errors that are prejudicial to the displacement uncertainty.

The principle of the proposed approach is to extend to the time domain the regularization strategy used spatially. If the velocity field evolves smoothly in time, the above discussion about the β parameter may be readily applicable to include the time dimension. Thus, for the same β value, small elements along the space direction(s) may still offer a good accuracy *provided* a sufficient number of images is considered along the time axis for each element. This temporal series may be used to compensate for the poor quality of each individual image.

Some approaches post-process *a posteriori* the measured velocity fields to extract, say, the coherent part of the latter [13] or to filter the measured data [14]. The objective of the present work is to propose an *a priori* approach in which a space–time decomposition is sought. The main advantage of the proposed method is the large number of pictures used that may allow one to reach the same uncertainty level with a small amount of spatial information, counter-balanced by a large amount of temporal data.

Sequences of images can be obtained from standard movies. To benefit from the large number of images, a temporal regularization

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doi:10.1016/j.optlaseng.2010.08.012

is called for. For instance, one may seek for steady-state velocity fields [15], or in the present case velocity fields that are decomposed over a set of piece-wise linear fields in space and time. This type of description is developed in the same spirit as global approaches [16,17], and in particular to finite-element based correlation algorithms whereby the displacement field is described by finite-element shape functions of the space variables [8,18].

Along those lines different strategies can be considered. The direct transposition of DIC is to search for displacement fields in space and time *simultaneously*. This route is not followed here since the sought fields are velocities and strain rates. It is well known that (time or space) derivatives will increase the noise level, and thus the displacement-formulated strategy may reveal unreliable. Therefore, the choice was made to focus directly on the velocity field as the main unknown to the problem. It will be shown that in spite of the fact that this velocity is the time derivative of displacement, good performances will be reached.

In the present case, a 2D approach is developed, namely, 1D in space and 1D in time. It is referred to as DIC applied to analyze movies (or Movie-DIC). The paper is organized as follows. First, the principle of the method is described. Then, artificial pictures are generated and the technique is carried out to determine *a priori* performances. Last, the spatio-temporal approach is applied to analyze the kinematics of a sample in a split Hopkinson pressure bar test.

2. Principle of the spatio-temporal analysis

The first step of the analysis consists in creating the so-called spatio-temporal map. For each picture, where x,y are the image coordinates, taken at several instants of time t, the gray level for a particular (chosen) position (x,y) is represented as a function of time t. Therefore, for a fixed y coordinate a sequence of images becomes an $f(x,t)$ map. The stacking principle is depicted in Fig. 1. The restriction to a single spatial coordinate (y being fixed) is suited for problems where the velocity is essentially along the x axis.

The measurement technique is based upon the conservation of the brightness [19,20]. The advection of the texture by a velocity field v (along the x-axis) is expressed as

$$f(x+v\,\mathrm{d}t,t+\mathrm{d}t)=f(x,t) \tag{1}$$

where the increment dt corresponds to one time interval between successive images (i.e. a "time pixel"). The aim is to estimate the velocity field $v(x,t)$ by using the brightness conservation. Minimization of the quadratic difference τ over space and time is used

$$\tau=\int_x\int_t [f(x,t)-f(x+v(x,t)\,\mathrm{d}t,t+\mathrm{d}t)]^2\,\mathrm{d}x\,\mathrm{d}t \tag{2}$$

The velocity field is decomposed over a basis of functions ϕ and φ as follows

$$v(x,t)=\sum_{i,j} a_{ij}\phi_i(x)\varphi_j(t) \tag{3}$$

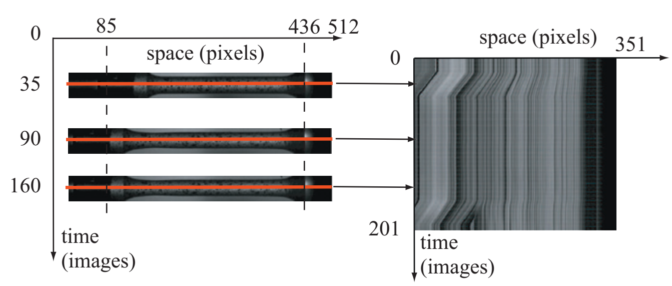

Fig. 1. Construction of a spatio-temporal map.

In the present case, finite-element shape functions are chosen, and their simplest form is adopted, namely, a piece-wise bi-linear description of the velocity field. However, it is conceivable to consider other sets of either continuous functions [15] or even discontinuous functions [21,22].

The proposed scheme is to solve this non-linear problem iteratively by a progressive adjustment of the velocity to the tangent linearized problem. The initialization of the unknown velocity field is here chosen to be equal to zero, $v^0(x,t)=0$. However, if a predetermination of the velocity field is available, it is straightforward to include it at this stage. The velocity $v^{(n+1)}(x,t)$ at step $n+1$ of this iterative scheme is determined from the Taylor expansion of the objective functional

$$\tau=\int_x\int_t [f(x^{(n)},t-\mathrm{d}t)-f_{,x}(x,t)(v^{(n+1)}-v^{(n)})\,\mathrm{d}t-f(x,t)]^2\,\mathrm{d}x\,\mathrm{d}t \tag{4}$$

with $f_{,x}(x,t)=\partial f(x,t)/\partial x$. In this expression $x^{(n)}$ is a short hand notation for the value x' such that $x'+v^{(n)}(x',t-\mathrm{d}t)\,\mathrm{d}t=x$. Note that the above expression is a specific choice out of many equivalent ones that differ only through second order terms. The advantage of this particular form is that the correction field is multiplied by $f_{,x}(x,t)$, which may be computed once for all iterations. This will ease the computational work as shown in the following.

One difficulty of the above approach, in particular for low quality images, is the use of a space derivative that may render the procedure sensitive to noise. A filtering of the images may be used. Note that in this case, the band filtering used in space and time should be adjusted so that their bounds are in proportion of the mean velocity.

The decomposition (3) is introduced in Eq. (4) and minimization with respect to a_{ij} leads to a linear system

$$\begin{aligned}\sum_{i,j}&\left(\int_x\int_t [\phi_i(x)\phi_k(x)\varphi_j(t)\varphi_l(t)f_{,x}^2(x,t)]\,\mathrm{d}x\,\mathrm{d}t\right)a_{ij}^{(n+1)}\\ &=\sum_{i,j}\left(\int_x\int_t [\phi_i(x)\phi_k(x)\varphi_j(t)\varphi_l(t)f_{,x}^2(x,t)]\,\mathrm{d}x\,\mathrm{d}t\right)a_{ij}^{(n)}\\ &\quad+\int_x\int_t (\phi_k(x)\varphi_l(t)f_{,x}(x,t)(f(x^{(n)},t-\mathrm{d}t)-f(x,t)))\,\mathrm{d}x\,\mathrm{d}t\end{aligned} \tag{5}$$

This elementary step is written in a compact form as

$$M_{ijkl}a_{ij}^{(n+1)}=B_{kl}^{(n)} \tag{6}$$

The reason for the specific choice made in Eq. (4) is now clear, namely, matrix **M** is computed once for all at the first iteration, and it does not depend on the current evaluation of the velocity field. However, the second member, **B** is dependent on $v^{(n)}$, but its evaluation is much less demanding computationally than **M**. At each step, the "deformed" image $f(x+v^{(n)}\,\mathrm{d}t,t+\mathrm{d}t)$ is corrected by using the velocity field estimate at the previous step in order to compute the second member. By inverting (6), the unknown degrees of freedom $a_{ij}^{(n+1)}$ are obtained, and thus the corresponding velocity field is estimated. Convergence, based on a measure of the norm of $a^{(n+1)}-a^{(n)}$, is reached in a few iterations (typically less than 10). By integrating the velocity field with respect to time, the displacement and thereafter the strain fields are obtained.

In order to validate the approach, the objective function is considered. Its value, normalized by the image size ($n_x\times n_t$),

$$R=\sqrt{\tau/(n_xn_t)} \tag{7}$$

gives the mean gray level difference of the matching of $f(x,t)$ with $f(x,t+\mathrm{d}t)$ using the measured velocity field. It is thus a *global* measure of the quality. Moreover, because τ is a space–time integral of the square of a residual field,

$$\delta\equiv\frac{1}{\Delta}|f(x,t)-v(x,t)\,\mathrm{d}tf_{,x}(x,t)-f(x,t+\mathrm{d}t)| \tag{8}$$