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# Analytical approximation of the InGaZnO thin-film transistors surface potential

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ABSTRACT

Surface-potential-based mathematical models are among the most accurate and physically based compact models of thin-film transistors, and in turn of indium gallium zinc oxide TFTs, available today. However, the need of iterative computations of the surface potential limits their computational efficiency and diffusion in CAD applications. The existing closed-form approximations of the surface potential are based on regional approximations and empirical smoothing functions that could result not accurate enough in particular to model transconductances and transcapacitances. In this work we present an extremely accurate (in the range of nV) and computationally efficient non-iterative approximation of the surface potential that can serve as a basis for advanced surface-potential-based indium gallium zinc oxide TFTs models.



## 1. Introduction

Amorphous indium gallium zinc oxide (a-IGZO) is a very promising candidate for the next generation thin-film transistors (TFTs) technology, thanks to the large electron mobility exceeding 10 $cm^2/V\,s$, optical transparency, mechanical flexibility, low temperature deposition, solution processability, good device stability [1,2]. In the near future, a-IGZO TFTs are expected to be used in high definition flexible displays, sensors, memories, and high-functionality circuits. The development of such applications requires accurate circuit simulation and extensive technology characterization that, in turn, relies on compact physically-based device models [3]. Surface potential based thin-film transistors models provide a consistent and accurate expression of currents and charges valid in all the operating regions [3–6]. They have emerged as a better alternative to the threshold voltage-based models as they are suitable for circuits at low power supply voltages and allow a deep physical modeling including the subthreshold region. Surface-potential-based models (SPBMs) represent an attempt to increase the physical content of compact device models, to reduce the complexity and the number of fitting parameters. On the other hand, the main drawback of SPBMs is the need for elaborate computations of the surface potential: it is usually calculated by an iterative solution of the well-known Gauss transcendental equation [3–7]. This procedure is computationally expensive, sometimes does not converge to the solution, and represents a significant detriment to implement SPBMs in popular circuit simulators.

Aim of this paper is to introduce a new closed-form analytical approximation of the surface potential that is computationally efficient, well behaved, and extremely accurate. Furthermore, the accuracy of the surface potential is a necessary but not sufficient condition for the utility of the analytical approximation. Indeed a set of stringent requirements is also imposed on MOSFET compact models. In particular, the partial derivatives of the surface potential should be continuous and smooth to compute transconductances and transcapacitances, traditionally a weak point of most analytic approximations. The analytical solution presented here differs from previous works in terms of accuracy and nature of the approximation, and satisfies the above requirements. Rather than matching two asymptotic approximations [3], we develop an approximate solution of the exact equation. The resulting expression is accurate in weak, moderate and strong accumulation over a wide range of physical parameters. The maximum absolute error is in the range of nV and the partial derivatives are continuous and smooth.

## 2. Surface potential

The conduction band edge of a-IGZO is composed of spherical overlapping orbitals that lead to a reduced density of band tail states in the amorphous phase. The conductivity of a-IGZO

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semiconductors is shown to be controlled by percolation and trap-limited conduction [8]. The density of band tail states influences the field effect mobility, while the average spatial coherence length and the potential fluctuation affect the percolation conduction. The density of tail states $g_t$ is conventionally assumed to be exponential [8]:

$$g_t = \frac{N_{T0}}{K_B T_T} \exp\left(\frac{E - E_C}{K_B T_T}\right) \tag{1}$$

where $N_{T0}$ is the concentration of tail localized states per unit volume, $T_T$ is the tail states characteristic temperature, $K_B$ is the Boltzmann constant, and $E_C$ is the conduction-band edge. The conductivity $\sigma$ of the a-IGZO, accounting for the percolation and the trap-limited conduction, reads [8]:

$$\sigma = q\mu_B N_C \exp\left(q\frac{\phi - V_{CH} - \phi_0}{K_B T}\right) \tag{2}$$

where $N_C$ is the concentration of states per unit volume in the transport band, $\phi$ the electrostatic potential, $V_{CH}$ the channel potential, $\phi_0 = E_G/2q$, $E_G$ is the band gap, $\mu_B = q\mu_0 \exp(-q\phi_{B0}/K_B T_T + q^2\sigma_B^2/2K_B T_T)$ is the band mobility $\mu_0$ modulated by the percolation term [8]. The surface potential $\phi_S$ is calculated by means of the Gauss equation at the semiconductor-insulator interface:

$$F_X = \frac{C_i}{\varepsilon_S}(V_{GS} - V_{FB} - \phi_S) \tag{3}$$

where $C_i$ is the gate capacitance per unit area, $V_{GS}$, $V_{FB}$ are the gate and the flat-band voltages, respectively, and $F_X$ is the electric field in the $x$-direction (normal to the channel).

Assuming the gradual channel approximation ($F_X \gg F_Y$), the electric field in the $x$-direction at the semiconductor–insulator interface reads:

$$F_X = \sqrt{\frac{2q}{\varepsilon_S}\int_{V_{CH}}^{\phi_S}(n + n_T)d\phi} = \sqrt{\frac{2}{\varepsilon_S}\left(K_B T_T N_T e^{q\frac{\phi_S - V_{CH} - \phi_0}{K_B T_T}} + K_B T N_C e^{q\frac{\phi_S - V_{CH} - \phi_0}{K_B T}} - \theta_{BLK}\right)} \tag{4}$$

where $\varepsilon_S$ is the semiconductor permittivity, $n_T = N_T \exp[(\phi - V_{CH} - \phi_0)/K_B T_T]$ is the trapped charge concentration, $n = N_C \exp[(\phi - V_{CH} - \phi_0)/K_B T]$ is the free charge concentration, $N_T = N_{T0}(\pi T/T_T)/\sin(\pi T/T_T)$, and $\theta_{BLK} = K_B T_T N_T e^{-q\frac{\phi_0}{K_B T_T}} + K_B T N_C e^{-q\frac{\phi_0}{K_B T}}$ is the bulk term, that is usually neglected [3].

## 3. Algorithm

To simplify the description and the coding of the approximate solution we define:

$$\omega(x) = \log\left[\frac{6}{5}\frac{x}{\log\left[\frac{12}{5}\frac{x}{\log[1+12x/5]}\right]}\right] \tag{5}$$

$$V_{GB} = V_{GS} - V_{FB}, \quad \xi = V_{CH} - \phi_0 \tag{6}$$

$$\Delta_F(x) = 2K_B\varepsilon_S T_{NC} e^{q\frac{x-\xi}{K_B T}} \tag{7}$$

$$\Delta_T(x) = 2K_B\varepsilon_S T_T N_T e^{q\frac{x-\xi}{K_B T_T}} \tag{8}$$

$$a(x) = \frac{q^2\Delta_F(x)}{2C_i^2 K_B^2 T^2} + \frac{q^2\Delta_T(x)}{2C_i^2 K_B^2 T_T^2} - 1 \tag{9}$$

$$b(x) = \frac{q\Delta_F(x)}{C_i^2 K_B T} + \frac{q\Delta_T(x)}{C_i^2 K_B T_T} + 2(V_{GB} - x) \tag{10}$$

$$c(x) = \frac{\Delta_F(x)}{C_i^2} + \frac{\Delta_T(x)}{C_i^2} - (V_{GB} - x)^2 - \frac{\theta_{BLK}}{C_i^2} \tag{11}$$

$$u(x) = \frac{-b(x) + \sqrt{b(x)^2 - 4a(x)c(x)}}{2a(x)} \tag{12}$$

then compute:

$$w_F = \left(\frac{q^2\varepsilon_S N_C}{2C_i^2 K_B T}\right)^{\frac{1}{2}} e^{q\frac{V_{GB} - \xi}{2K_B T}} \tag{13}$$

$$w_T = \left(\frac{q^2\varepsilon_S N_T}{2C_i^2 K_B T_T}\right)^{\frac{1}{2}} e^{q\frac{V_{GB} - \xi}{2K_B T_T}} \tag{14}$$

$$s_F = V_{GB} - 2\frac{K_B T}{q}\omega(w_F) \tag{15}$$

$$s_T = V_{GB} - 2\frac{K_B T_T}{q}\omega(w_T) \tag{16}$$

$$\eta_0 = \mathrm{Min}[s_F, s_T] \tag{17}$$

$$\eta_1 = \eta_0 + u(\eta_0) \tag{18}$$

and finally the surface potential reads:

$$\phi_S = \eta_1 + u(\eta_1) \tag{19}$$

The algorithm can be summarized as follows. First the surface potentials accounting for the band or the tail states only, $s_F$ and $s_T$ are calculated by means of Eqs. (15) and (16). The function $\omega(x)$, in fact, is a simple yet accurate approximation of the Lambert function for real values [9] with a maximum relative error of 2.39%. In Eq. (17), $s_F$ and $s_T$ are chosen so that $\eta_0$ represents a crude initial approximation of $\phi_S$ based on a single exponential DOS, i.e. the band or tail states only. In fact, if a single exponential DOS is accounted for, by the way of example the trapped charge only, the Gauss equation reads:

$$\frac{C_i}{\varepsilon_S}(V_{GS} - V_{FB} - \phi_S) = \sqrt{\frac{2K_B T_T N_T}{\varepsilon_S}\left(e^{q\frac{\phi_S - V_{CH} - \phi_0}{K_B T_T}} - e^{-q\frac{\phi_0}{K_B T_T}}\right)} \tag{20}$$

Furthermore, observing that $\exp(-q\phi_0/K_B T_T) \cong 0$, Eq. (20) can be analytically solved by means of the well-known Lambert function $W(x)$ and the surface potential accounting for the tail states only, $\phi_{Stail}$, reads:

$$\phi_{Stail} = V_{GB} - 2\frac{K_B T_T}{q}W(w_T) \tag{21}$$

Unfortunately, the Lambert function is seldom available in the CAD software and is computationally expensive. Hence, we use the approximate expression of Eq. (5) $\omega(x) \cong W(x)$; $\omega(x)$ is computationally efficient (it requires the evaluation of three logarithmic functions only), continuous, and accurate enough to compute $s_T \cong \phi_{Stail}$. At this stage, in fact, the approximation introduced by neglecting the free charges in the Gauss equation is more severe than the error introduced by $\omega(x)$. Same considerations hold for $s_F$. Then, the single exponential initial guess $\eta_0$ is computed thanks to $s_T$, $s_F$ (Eq. (17)). In Fig. 2, the idea underneath the approximation $\eta_0 = \mathrm{Min}[s_F, s_T]$ is shown: one can see at a glance that $\phi_S$ can be roughly approximated by $s_T$ if $s_T < s_F$, and by $s_F$ if $s_F < s_T$. It is a crude approximation but accurate enough as initial guess.