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# Diffusion studies in transparent liquid mediums utilizing polarization imaging

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ABSTRACT

Diffusion process occurs in many areas of physical, chemical and biological sciences. The imaging/monitoring of this transfer process and the measurement of diffusion coefficient are of utmost importance, in these areas of research. Usually interferometric methods are used for this. Even though very accurate, they require controlled environments (especially to be isolated from external noise) and should adhere to stringent optical considerations. Single-beam optical techniques are more suitable in noisy environments. A ray passing through a non-uniform refractive index distribution deflects towards region of higher refractive index. A diffusing medium has such a non-uniform refractive index distribution. If this deflection can be measured somehow, it can be used to find the refractive index gradient and hence the refractive index distribution inside the medium. Here a method is proposed to measure these deflections and hence the diffusion coefficient using active optical elements, by converting the incident light into a spatially varying polarization pattern. The method is demonstrated using optical simulations.



## 1. Introduction

Diffusion is nothing but the migration of molecules due to a concentration gradient. This process will continue till the existing concentration gradient is nullified. Diffusion occurs in diverse fields from chemical engineering, biology, pollution control to separation of isotopes [1,2]. Monitoring and measurement of diffusion is of great importance and is also a very challenging as well as interesting problem. Optical techniques are obviously one of the best methods to probe the process of diffusion in transparent mediums. Many optical methods can be found in the literature for this purpose [3–13]. These methods range from conventional interferometry, holographic interferometry to speckle pattern interferometry. But most of the interferometric methods are two beam techniques, needing more optical elements, stringent optical setups as well as the exact adjustment of beam ratios for high contrast fringes. So there is scope for new innovative methods to monitor diffusion process. Since the diffusing system is a non-uniform one, other than the phase change due to the time delay, one will encounter beam deflection, when the concentration gradients are appreciable. This deflection contains information about the refractive index gradient. If this deflection can be measured, it will shed light on the state of the diffusing medium.

The output spatial distribution of polarization of a linearly polarized incident beam on a birefringent or optically active crystal will depend upon the distance the incident beam passes through the crystal. This aspect could be utilized to construct a polarization detecting wavefront sensor [14,15]. This type of sensor can then be used to determine or monitor diffusion process in transparent mediums. This method is described in this paper along with the simulated results. In the simulations a birefringent crystal is considered in the wavefront sensor.

## 2. Experimental realization and theory

Fig. 1 shows the schematic of the experimental setup. Light from the source (it is not necessary to have temporal coherent), is collimated. This collimated beam then passes through the diffusing solution. The output from the diffusion cell is linearly polarized and allowed to pass through birefringent crystal. The output spatially varying polarization pattern from the crystal is converted into an intensity pattern using an analyzer. This intensity pattern is then imaged on to a CCD camera.

The basic property of a birefringent crystal with its optics axis not oriented parallel to the beam propagation direction is that it

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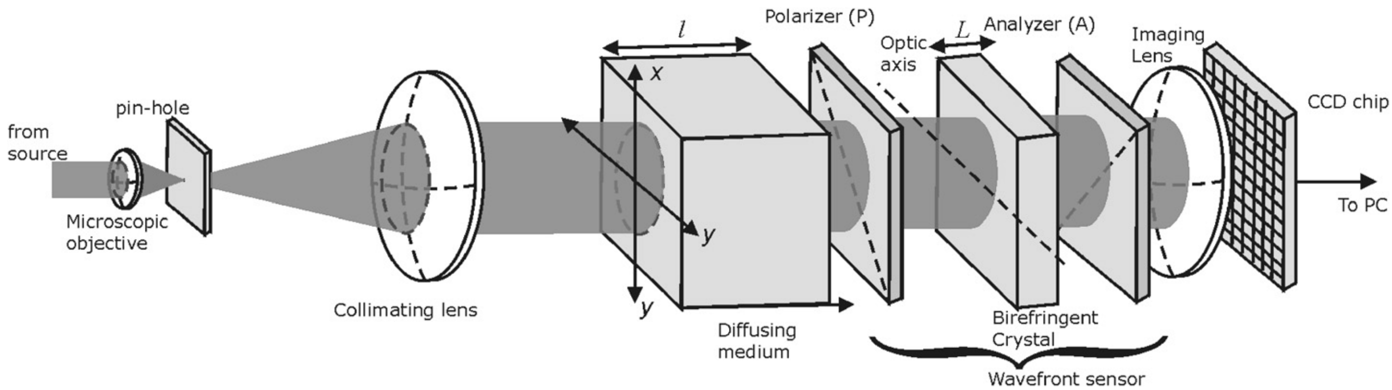


**Fig. 1.** Experimental setup.

splits the incident light into ordinary and extra ordinarily polarized components. These two components will then travel with different velocities through the crystal, since they encounter different refractive indices. These waves will combine at the output face of the crystal and the resulting polarization of the wave depends upon the propagation length of the waves inside the crystal. If the wavefront is not collimated, the output from the crystal will have a spatially varying polarization pattern. This spatial variation of the polarization pattern is therefore a characteristic of the incident wavefront, which is modulated by the medium through which it passes. The output polarization state is therefore a reflection of the phase changes induced to different portions of the wavefront because of the refractive index gradient existing in the modulating medium.

The phase difference ($\Delta\varphi$) between the ordinary and extraordinary rays at the output face of the birefringent crystal of thickness $L$ is given by [16]

$$\Delta\varphi = \frac{2\pi}{\lambda}(|n_e - n_o|)\frac{L}{\cos(\theta)} \tag{1}$$

where $\lambda$ is the vacuum wavelength of the incident light, $n_o$ and $n_e$ are the refractive indices encountered by the ordinary and extra ordinary rays and $\theta$ depends upon the angle of incidence at the birefringent crystal of the signal beam. The output intensity ($I$) from the analyzer ($A$), with its direction of polarization rotated 90° to the input polarizer and for a beam polarized at 45° to the optic axis, is given by

$$I = I_0 \sin^2\left(\frac{\Delta\varphi}{2}\right) \tag{2}$$

where $I_0$ is the intensity at the output face of the birefringent crystal. From Eq. (1), it can be seen that as the propagation distance inside the birefringent medium increases, the phase difference between the o- and e-rays increases, thereby increasing the rotation of plane of polarization of that portion of the wavefront.

In the diffusion cell, heavier concentration solution is introduced below the lighter concentration solution using a capillary tube mechanism. It is assumed that the process of diffusion starts after the cell is filled. Non-steady state diffusion process is given by Fick's second law and for one-dimensional diffusion (say for example in $x$-direction), its solution at time instant $t$ is [17]

$$C(x,t) = \frac{(C_1 + C_2)}{2} + \frac{(C_2 - C_1)}{\sqrt{\pi}}\int_0^{x/\sqrt{4Dt}} e^{-\eta^2}\, d\eta \tag{3}$$

where $C_1$ and $C_2$ are the concentrations of the lighter and heavier solutions respectively initially separated at position $x = 0$, $D$ is the diffusion coefficient and the integral is nothing but the error function [18] of $x/\sqrt{4Dt}$. For small concentration range, the

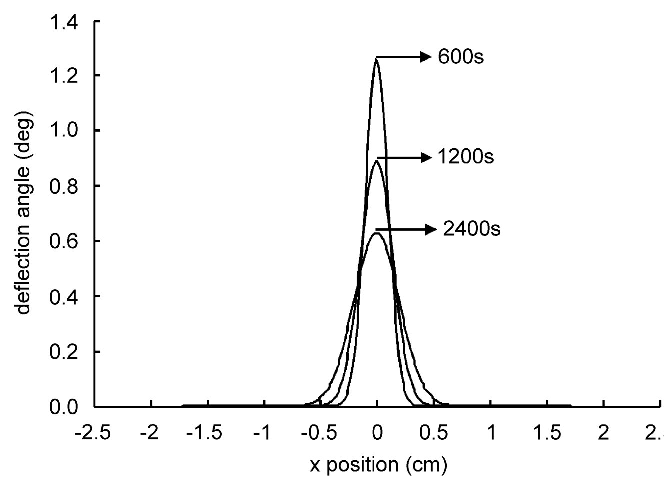


**Fig. 2.** Change in angle of deflection with position at different time instances. Ammonium dihydrogen phosphate solution of average concentration 0.4981 M was considered.

refractive index can be expressed as a linear variation of concentration as

$$n(x,t) = mC(x,t) + n_c \tag{4}$$

where $m$ is the derivative of refractive index with concentration for the range existing inside the diffusion cell and $n_c$ is a constant. The refractive index distribution inside the diffusion cell is non-uniform (Eq. (4)) resulting in the deflection of light rays entering the cell and the amount of deflection will depend on the refractive index gradient [19]. The angle through which a ray entering at $z = 0$ will be deflected is

$$\theta(x,t) = \frac{dx}{dz} = \frac{l}{n}\frac{\partial n(x,t)}{\partial x} = \frac{l}{n}m\left(\frac{C_1 - C_2}{2}\right)\frac{\exp(-x^2/4Dt)}{\sqrt{\pi Dt}} \tag{5}$$

where $l$ is the width of the diffusing medium. The change in deflection angle along the diffusion direction for different time instances is shown in Fig. 2 ($C_1 = 0.4484$ M, $C_2 = 0.5497$ M, average concentration $C_{avg} = 0.4981$ M of ammonium dihydrogen phosphate). The deflection will decrease with time as the refractive index gradient decreases. When $t$ is very large the refractive index gradient becomes zero and the diffusion medium, will not deflect the incident beam anymore. It can also be seen from Fig. 2 that, the deflection is appreciable early in the diffusion process. Therefore the imaging of the deflection at these time instances would give more accurate results.