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Correlated noise in bipolar transistors: Model implementation issues

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ABSTRACT

A new orthogonalization scheme is suggested for implementing correlated noise of bipolar transistors. The scheme provides a necessary condition on the non-quasi-static (NQS) models that can be used to obtain an implementation-suitable correlated noise model. One of the solutions presented here corresponds to a single node realization not reported so far. The g_m-factor is introduced in the noise analysis explaining the deviations of a former noise model from device simulations. The model is extended to include the collector space-charge-region induced noise by retaining the simplicity of the realization and preserving the model parameter count.

1. Introduction

SPICE treats noise fluctuations in AC analysis as deterministic signals. It is possible since in a small relative bandwidth, $\Delta f/f_0$ around a center frequency f_0, the stochastic noise fluctuations can be approximated by deterministic harmonic functions [1]. However, unlike deterministic signals, noise signals are summed by their powers. Hence, a pair of correlating noise sources of spectral amplitudes μ_1, μ_2 in the network produces the following power spectral density (PSD) at one of the ports

$$\begin{aligned} S_{port} &= \langle (h_1 \cdot \mu_1 + h_2 \cdot \mu_2)(h_1 \cdot \mu_1 + h_2 \cdot \mu_2)^* \rangle \\ &= S_{11}|h_1|^2 + S_{22}|h_2|^2 + 2\Re\{S_{12} \cdot h_1 \cdot h_2^*\}. \end{aligned} \tag{1}$$

with $h_{1,2}$ as the complex network functions from a particular source to the selected port, while the angle brackets $\langle .. \rangle$ stand for the ensemble average of the randomly varying spectral amplitude [2]. For convenience unit bandwidth $\Delta f = 1$ Hz has been assumed here and the PSD-s are denoted as $S_{ij} = \left\langle \mu_i \cdot \mu_j^* \right\rangle / \Delta f = \left\langle \mu_i \cdot \mu_j^* \right\rangle$. The shortcoming of SPICE is that it omits the last term in (1), thereby, discarding the influence of noise correlation altogether [3]. A solution to this problem is to construct an equivalent system using uncorrelated sources as proposed in [4]. A version of this concept reported independently in [5] has limited applicability since it assumed noise signals of equal spectral amplitudes. Moreover it can be used only at a fixed frequency. However, proper guidelines are laid down in [5] for correlated noise implementation in Verilog-A. The diagonalization method of [4] was applied in [6] to implement noise correlation in the HICUM model [7]. Simulation results, however, led to excessively large noise levels in some cases [8] due to an erroneous ω^4 term. The improvement claimed in [9] used eight extra nodes and the theory behind the solution was not reported. Though the ω^4 term was removed in [10], an erroneous assumption on the correlation between the noise of the base current I_{BEi} and that of the collector current I_T [11] (see Fig. 1) was carried over from [6]. Such an assumption is nonphysical for narrow base transistors as pointed out in [12]. However, this misconception did not affect the implementation of [10] and to the best of the authors' knowledge, this has been the first flawless implementation of the correlated noise for bipolar transistors. A large variety of spectral densities for the noise models can be found in the literature. For example, the base spectral density was arbitrarily selected in [13] just to ensure a physical correlation coefficient. Other works [14–16] suggested an empirical noise delay time to account for the frequency dependence in the base noise PSD ($S_{i_{nB}}$). Empirical co-factors were assigned to the various PSDs in [17] requiring measurements to extract them. An overview on the most recent descriptions of the noise spectral densities can be found in [18].

In this paper we investigate the conditions on the noise spectral densities that can be realized in SPICE. The flexibility of a novel orthogonalization scheme, introduced here to carry out the investigation, allows us to propose a single-node implementation of correlated noise.

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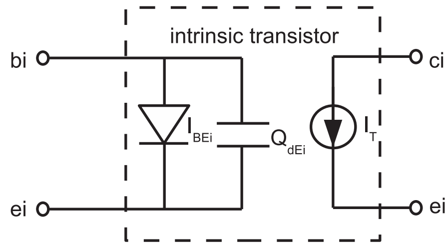

Fig. 1. Common emitter equivalent of van der Ziel and Bosman [12].

2. SPICE modeling of correlated noise

2.1. An extended orthogonalization scheme

When there are several noise sources in a network, the transfer functions and excitations can be arranged in column vectors $\boldsymbol{h}$ and $\boldsymbol{\mu}$, respectively, converting (1) to

$$S_{port} = \left\langle (\boldsymbol{h}^T \cdot \boldsymbol{\mu})(\boldsymbol{h}^T \cdot \boldsymbol{\mu})^H \right\rangle = \boldsymbol{h}^T \cdot \boldsymbol{S}_\mu \cdot \boldsymbol{h}. \tag{2}$$

Superscripts T and H stand for the transpose and the Hermitian operator and $\boldsymbol{S}_\mu = \langle \boldsymbol{\mu} \cdot \boldsymbol{\mu}^H \rangle$ denotes the PSD matrix of the noise sources [19]. Since a network connected to the input port of a controlled source is invisible from its output side, regarding the elements of $\boldsymbol{\mu}$, controlled sources do not modify the transfer functions $\boldsymbol{h}$. Hence, we can assume $\boldsymbol{\mu}$ to be a linear combination of new noise sources $\boldsymbol{\lambda}$ as

$$\boldsymbol{\mu} = \boldsymbol{A} \cdot \lambda. \tag{3}$$

The elements of the transfer matrix $\boldsymbol{A}$, in general, represent sub-circuits of passive concentrated elements and controlled sources. Substituting (3) in (2) yields

$$S_{port} = (\boldsymbol{h}^T \cdot \boldsymbol{A}) \cdot \langle \lambda.\lambda^H \rangle \cdot (\boldsymbol{h}^T \cdot \boldsymbol{A})^H = (\boldsymbol{h}^T \cdot \boldsymbol{A}) \cdot \boldsymbol{S}_\lambda \cdot (\boldsymbol{h}^T \cdot \boldsymbol{A})^H. \tag{4}$$

The elements of $(\boldsymbol{h}^T \cdot \boldsymbol{A})$ are the augmented transfer functions between the new λ_i sources and the selected port while the $\boldsymbol{S}_\lambda$ denotes the PSD matrix of the new noise sources. Particularly when $\boldsymbol{S}_\lambda$ is a diagonal matrix, the modified network defined by (4) is formally suited in SPICE implementation. For example, the decomposition in [4,10] directly focuses on the orthogonalization of the PSD matrix. This is achieved by a projection with a triangular matrix containing unities in the main diagonal. The cost of such a solution is that the primarily uncorrelated sources are strictly constrained. For a more flexible orthogonalization scheme, we restrict the solution for only two correlated sources that will allow an arbitrary selection of the primary sources. The composite noise spectral amplitudes will be a linear combination of the primarily uncorrelated spectral amplitudes with coefficients representing circuit elements. The difference of such a scheme from the PSD oriented one of [4,10] makes it convenient to apply the network theoretical principles to solve noise related problems.

From (3) one obtains

$$\boldsymbol{S}_\lambda = \boldsymbol{A}_{inv} \cdot \boldsymbol{S}_\mu \cdot \boldsymbol{A}_{inv}^H \tag{5}$$

with $\boldsymbol{A}_{inv}$ as the inverse of matrix $\boldsymbol{A}$. In case of two noise sources, an orthogonal $\boldsymbol{S}_\lambda$ results into $\langle \lambda_1 \cdot \lambda_2^* \rangle = \langle \lambda_1 \cdot \lambda_2^* \rangle^* = 0$ yielding

$$\begin{aligned} &(-a_{22}S_{11} + a_{12}S_{12}^*)a_{21}^* + (a_{22}S_{12} - a_{12}S_{22})a_{11}^* = 0 \\ &\left[(-a_{22}S_{11} + a_{12}S_{12}^*)a_{21}^* + (a_{22}S_{12} - a_{12}S_{22})a_{11}^*\right]^* = 0. \end{aligned} \tag{6}$$

Here a_{ij} represents the elements of matrix $\boldsymbol{A}$. Using (6), the main diagonal elements of $\boldsymbol{S}_\lambda$ read

$$\langle \lambda_1 \cdot \lambda_1^* \rangle = \frac{a_{22}S_{11} - a_{12}S_{12}^*}{a_{11}^* \cdot det(\boldsymbol{A})} \tag{7}$$

$$\langle \lambda_2 \cdot \lambda_2^* \rangle = \frac{a_{11}S_{22} - a_{21}S_{12}}{a_{22}^* \cdot det(\boldsymbol{A})}. \tag{8}$$

Being conjugates, conditions given in (6) make only one constraint; therefore, one can arbitrarily select $a_{21} = 0$ or $a_{12} = 0$. This results in the following two sets of solutions for the coefficients

$$\begin{aligned} &|a_{11}|^2 = \frac{det(\boldsymbol{S})}{\langle \lambda_1 \cdot \lambda_1^* \rangle \cdot S_{22}}, \quad a_{12} = a_{22}\frac{S_{12}}{S_{22}}, \\ &a_{21} = 0, \qquad\qquad |a_{22}|^2 = \frac{S_{22}}{\langle \lambda_2 \cdot \lambda_2^* \rangle}; \end{aligned} \tag{9}$$

and

$$\begin{aligned} &|a_{11}|^2 = \frac{S_{11}}{\langle \lambda_1 \cdot \lambda_1^* \rangle}, \quad a_{12} = 0, \\ &a_{21} = a_{11}\frac{S_{12}^*}{S_{11}}, \qquad |a_{22}|^2 = \frac{det(\boldsymbol{S})}{\langle \lambda_2 \cdot \lambda_2^* \rangle \cdot S_{11}}. \end{aligned} \tag{10}$$

The coefficient matrices (9) and (10) yield two independent orthogonal decomposition for (3).

2.2. A circuit realization constraint

According to circuit theory [20], the network functions of the elements of $\boldsymbol{A}$ must be rational fractional functions with Hurwitz polynomials $H(s)$ in the denominator. The numerator polynomials can be combined with the transfer factors of the controlled sources resulting in the complex polynomials $t(s)$. This implies the following composition for the elements of $\boldsymbol{A}$

$$a_{ij} = \frac{t_1(s)}{H_1(s)} + \frac{t_2(s)}{H_2(s)} + \cdots \tag{11}$$

Merging (11) in a single fraction, the common denominator still remains Hurwitz type; hence, a_{ij} must not contain poles in the right half plane. Therefore, the realization of a_{ij} requires the *necessary condition*, that the denominator of a_{ij} must be a Hurwitz polynomial. The violation of this condition makes a_{ij} non-realizable or unconditionally unstable.

2.3. Noise signal propagation in SPICE

As seen from (1) the noise spectral amplitudes are not needed in the calculation of the port PSDs; hence, SPICE – along with Verilog-A – requires only the PSDs and the network locations of the sources. Keeping this in mind, a source of noise spectral amplitude, μ is marked by its PSD, S_μ in the schematics (Fig. 2) while the terms in the augmented transfer functions $\boldsymbol{A}$ are represented by conventional circuit elements.

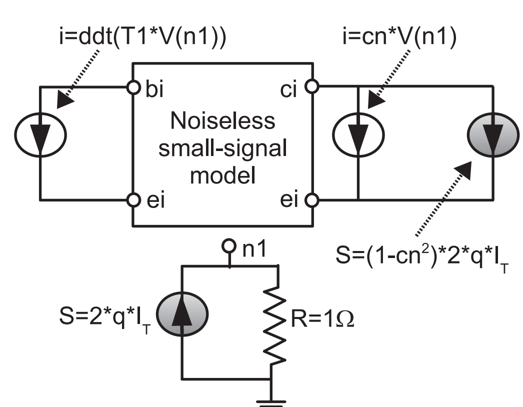

Fig. 2. SPICE implementation of BJT correlated noise.