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Compact model for non-local avalanche effect in advanced bipolar transistors: An assessment of the relaxation length and its temperature dependence

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ABSTRACT

We present a physics based compact model formulation for non-local avalanche effects. It is explicit and in terms of elementary functions, hence suitable for implementation in existing compact transistor models. The formulation has only two material coefficients as parameters: the energy relaxation length and its temperature coefficient. We present a detailed verification of our model against measured avalanche characteristics, as a function of both bias and temperature, for Si and SiGe industrial bipolar transistors. We demonstrate that the model is complete and accurate enough for the parameter extraction to be taken as an in situ measurement for both the electron energy relaxation length and its temperature coefficient: values obtained correspond to the values published earlier in the semiconductor literature.

1. Introduction

To meet the requirements of emerging RF applications, in modern bipolar transistors breakdown voltages are aggressively traded off against speed performance. Fortunately, RF SiGe HBTs explore highly doped collector base junctions in which the electric field $E(x)$ is sufficiently peaked to relax this trade off, to some extent, by enhancing the breakdown voltages through the so-called non-local avalanche effect [1,2]. Nonetheless, circuit designers are forced to exploit transistor operation above open base common-emitter breakdown voltage limits. At the same time, either because of application determined requirements (e.g., tolerances with respect to possibly extreme environmental conditions, such as in automotive applications) or because of significant self-heating in SiGe HBT applications, modeling of temperature dependencies with respect to all transistor characteristics requires undiminished attention. Hence, firstly, the need [3] for adequate compact model formulations of non-local avalanche and, secondly, the motivation to include in these, the modeling of temperature dependencies [4].

In this paper, we shall present a physics based compact model formulation for non-local avalanche, including its temperature dependence. Our formulation is explicit and in terms of elementary functions, hence fit for incorporation in standard compact transistor models; in this our solution is distinct from more computationally demanding approaches, e.g. [5,6].

Complementary to developing a compact model formulation, the present work includes an experimental assessment of the significance of its assumed device physical basis. The starting points for our derivation are the approximate energy balance equation for electron temperature [1,2] and Chynoweth's empirical law for the ionization coefficient [7]. Our compact model parameters are the energy relaxation length λ_e and its temperature coefficient A_{λ_e}. The experimental verification of the device physical basis of our model then consists of a demonstration that careful extraction from the bias and temperature dependence of industrial bipolar transistor weak avalanche characteristics, yields in situ values of λ_e and A_{λ_e} that match earlier published values [1,4] for these material coefficients. This extraction includes careful modeling of independent physical effects that interfere with avalanche in the weak-avalanche regime, so that the study presented can be taken as an experimental assessment of the device physical effects that are significant in the weak avalanche regime, in present day industrial bipolar transistors.

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2. Compact expression for non-local avalanche

When electrons in the conduction band of a semiconductor device carry a drift current under the influence of an electric field $E(x)$, the elevation $\Delta T_e(x) = T_e(x) - T_0$ of the electron temperature $T_e(x)$ with respect to the equilibrium lattice temperature T_0 is governed by the following approximate energy balance equation [1,8]

$$\frac{d\Delta T_e(x)}{dx} + \frac{1}{\lambda_e}\Delta T_e(x) + \frac{2q_e}{5k_B}E(x) = 0. \tag{1}$$

In this expression, k_B is Boltzmann's constant, q_e is the elementary charge, and λ_e the energy relaxation length.

For spatially homogeneous collector doping concentration N_{epi}, the electric field $E(x)$ (where $E(x) < 0$) throughout the depletion region in the collector region of a bipolar transistor takes the form $E(x) = ax + E_0$. The gradient of the field a depends on the doping N_{epi} and on the collector current. Under the assumption of constant λ_e and initial condition $T_e(x) = T_0$, Eq. (1) then yields

$$\Delta T_e(x) = \frac{2q_e\lambda_e}{5k_B}\left[(a\lambda_e - E_0)\left(1 - \exp\left(-\frac{x}{\lambda_e}\right)\right) - ax\right]. \tag{2}$$

To represent non-local impact ionization, following [1] we express Chynoweth's empirical law [7] for the impact-ionization coefficient α as a function of $\Delta T_e(x)$

$$\alpha[\Delta T_e(x)] = A_n \exp\left[-\frac{2q_e\lambda_e}{5k_B}\frac{B_n}{\Delta T_e(x)}\right]. \tag{3}$$

In this expression, the quantities A_n and B_n are the original coefficients of Chynoweth's law; the values of these material parameters are experimentally well-established [9–11].

The weak avalanche current $I_{\text{avl}} = (M_n - 1)I_C$ follows from the collector current I_C through the global multiplication factor $(M_n - 1)$, which is defined as the integral of (3) over the collector-epilayer width W_{epi}:

$$(M_n - 1) = \int_0^{W_{\text{epi}}} \alpha[\Delta T_e(x)]dx. \tag{4}$$

In any realistic regime of parameters, as valid for conditions in the collector of bipolar transistors, the function $\alpha[\Delta T_e(x)]$, turns out to be a sharply peaked function about the position $x_{\max}$ of the extreme value of the electron temperature, which follows from expression (2) as:

$$x_{\max} = \lambda_e \ln\left[1 - \frac{E_0}{a\lambda_e}\right]. \tag{5}$$

Taking advantage of the fact that the function $\alpha[\Delta T_e(x)]$ is sharply peaked and rapidly approaches zero away from position $x_{\max}$, the integral (4) of (3) on basis of (2) can be approximated by means of a Taylor series expansion of the integral in terms of the width of the peak (see Appendix A). It is assumed that in the context of embedding into a full compact transistor model, an expression for the relevant electric field of the form $E(x) = ax + E_0$ is available; at position $x_{\max}$ this electric field has the value

$$E_{\text{avl}} = ax_{\max} + E_0. \tag{6}$$

With expression (A.5) from Appendix A, we then arrive at the following closed form approximation of multiplication factor (4):

$$(M_n - 1) \approx \frac{4\gamma_f E_{\text{avl}} A_n \lambda_e \exp(B_n/E_{\text{avl}})}{E_{\text{avl}} - \sqrt{6a\lambda_e(2E_{\text{avl}} + B_n) - E_{\text{avl}}^2}}. \tag{7}$$

In practice, a value of the constant coefficient γ_f can be found, once and for all, by comparing the results for $(M_n - 1)$ obtained by (7) with those obtained by numerical integration of relation (4). As an illustration, the value for γ_f suitable for integration of Gaussian distributions is given in the appendix. With respect to integration of the class of functions (3) and in the context of our present application, we found $\gamma_f = 1.645$ to be adequate.

The only new physical parameter introduced by expression (7) is the relaxation length λ_e. Following [4], we model its dependence on ambient temperature T by a power law as

$$\lambda_e(T) = \lambda_e(T_{\text{ref}})\left(\frac{T}{T_{\text{ref}}}\right)^{A_{\lambda_e}} \tag{8}$$

where T_{ref} denotes a reference temperature. Relation (8) is based on the relation $\lambda_e = \frac{5}{3}\tau_e v_{\text{sat}}$ [1], in which τ_e denotes the energy relaxation time and v_{sat} the saturated drift velocity. Because the dependence of τ_e on ambient (i.e. lattice) temperature[12] is much weaker than that of v_{sat} [13,14], the temperature scaling of λ_e will resemble the temperature scaling of v_{sat}, which typically [13,14] is modeled as a power law of the form (8). Therefore, the value of A_{λ_e} should closely approximate the corresponding parameter for the temperature dependence of v_{sat} [13,14].

3. Implementation in Mextram

In industrial applications, compact transistor model expressions occasionally become subject to evaluation with parameter and bias values outside their range of physical validity. One instance of this regarding (7) would be evaluation in the high current regime. In a real bipolar device, the slope a of the electric field then could vanish or even change sign due to the Kirk effect [15]. In the context of (7), this would invoke implementation issues, as the argument of the square root function would become negative.

With respect to this issue we follow [3] in the viewpoint that application of (7) in combination with reversal of the sign of a is actually unrealistic. In the present paper therefore, we shall restrict ourselves to developing a counterpart of (7) that will provide a good approximation in the low and intermediate current regimes, while it will be such that I_{avl} will become negligible in the high current limit. Shortly, the modifications we shall apply will be such that they preserve accuracy of the formulation for low and intermediate collector currents, but quench the avalanche current in the high current limit, so as to make the model formulation globally robust.

In the regime of low and intermediate currents, the term $-E_{\text{avl}}^2$ contributes only a percentage of the full argument of the square root function. As a first step towards eliminating the possibility of sign reversal of the square root term, we approximate the argument of the root function by dropping the term $-E_{\text{avl}}^2$. To ensure that the remaining argument of the root function shall never vanish, we replace the terms E_{avl} by a counterpart $\widehat{E}_{\text{avl}}$, which is smoothly bounded from below by a limiting value $-0.5B_n$. Implementation of smoothly bounded functions is a standard technique in the context of compact modeling [15]. The result for the new expression for (7) is:

$$M_n - 1 \approx \frac{4\gamma_f \widehat{E}_{\text{avl}} A_n \lambda_e \exp(B_n/\widehat{E}_{\text{avl}})}{\widehat{E}_{\text{avl}} - \sqrt{6a\lambda_e(2\widehat{E}_{\text{avl}} + B_n)}}. \tag{9}$$

For robustness, it then would suffice that a be strictly positive. In terms of device physics, this comes down to using a compact model formulation of the electric field that in the high current limit quenches the sign reversal of a due to the Kirk effect. In, for example, the Mextram 504 [15] bipolar transistor model, such a formulation of the electric field is available. Indeed in Mextram, a can be calculated as E_M/λ_D, where E_M and λ_D are both strictly