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Research article

# Measuring the sustainability of a natural system by using multi-criteria distance function methods: Some critical issues

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ABSTRACT

There is an important body of literature using multi-criteria distance function methods for the aggregation of a battery of sustainability indicators in order to obtain a composite index. This index is considered to be a proxy of the sustainability goodness of a natural system. Although this approach has been profusely used in the literature, it is not exempt from difficulties and potential pitfalls. Thus, in this paper, a significant number of critical issues have been identified showing different procedures capable of avoiding, or at least of mitigating, the inherent potential pitfalls associated with each one. The recommendations made in the paper could increase the theoretical soundness of the multi-criteria distance function methods when this type of approach is applied in the sustainability field, thus increasing the accuracy and realism of the sustainability measurements obtained.

## 1. Introduction

The origin of the concept of sustainability has been traced by Pretzch (2014) referring to key research undertaken by Central European foresters more than 300 years ago (von Carlowitz, 1713). The basic idea of these pioneers was to request from a forest a long-term stable supply of a flow of goods and services necessary for the welfare of human beings. Later on, this concept of sustainability was transferred from forestry to other natural systems. Until very recently the commodities and services demanded from natural systems was always understood as being within a context of mono-functionality. That is, the basic purpose of natural systems is to provide inputs that the production system transforms into goods and services, whose consumption satisfies primary human needs.

The above approach for measuring the sustainability of a natural system was not questioned until the second part of the last century, when it was recognized that the environment had physical limits, making it necessary to move to a new context of multi-functionality. In this direction the seminal work by Gregory (1955) should be cited. Although this work is focused on forest systems, its basic ideas are straightforwardly applied to the multi-functionality of any natural systems. In fact, modern societies require not only the long-term durable provision throughout the time of raw materials from natural systems to be transformed into outputs, but also a supply of an important number of environmental goods and services like biodiversity conservation, carbon sequestration, soil erosion, etc.

The above mentioned shift towards the multi-functionality of natural systems implies key changes in the concept and measurement of sustainability. In other words, nowadays any specific natural system should supply a lasting broad spectrum of goods and services of a very different nature. Some of them are valued by the markets, but others, although no less essential, have no market value.

Within this new scenario it seems sensible to use a battery of indicators as a proxy of the different functions provided by a natural system. Thus, any sustainability measurement is established by the aggregation of those indicators into a single composite index, the value achieved by this index being a proxy of the respective sustainability goodness.

Following the above direction, a promising line of research is to treat the indicators as criteria. In this way, it is possible to resort to

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the work done in the Multiple Criteria Decision Making (MCDM) field. In fact, the purpose of MCDM is to aggregate in one way or another a set of conflicting criteria, so that it is only needed to change criteria for indicators and to proceed to their aggregation. Given the correspondence between the aggregation of criteria and that of sustainability indicators, in the last decades a substantial body of literature applying MCDM techniques to the measurement of sustainability has appeared. In a recent review (Diaz-Balteiro et al., 2017a) more than 270 papers included in the ISI Web of Science database dealing with sustainability from a multi-criteria perspective have been found, with a considerable increase in papers published in recent years.

Among the different MCDM approaches used for addressing sustainability issues, the multi-criteria methods based on the minimization of distance functions (MDF) have achieved paramount importance (Lozano-Oyola et al., 2012). However, despite the interest and success of this orientation, many of the applications reported in the literature are not exempt from difficulties, which imply poor modeling practices, leading to possibly erroneous results. The main aim of this paper is to detect several types of vital issues regarding the application of different MDF techniques for measuring natural system sustainability. Besides this, several methods to overcome these potential insufficiencies are proposed. The practical effectiveness of this type of approach could therefore be increased. It should be noted that the idea of this manuscript was not to review MDF methods applied in sustainability issues, but to make some reflections on the use of these methods, in order to gain some insight and make recommendations for future work.

The paper is organized as follows: in Section 2 the basic aspects of MDF methods are presented; after that, a set of major issues is given, proposing methods for overcoming possible misconceptions and pitfalls. The paper ends up raising some basic conclusions.

## 2. Multi-criteria distance function models: some basic aspects

In this section, the core of the multi-criteria distance function (MDF) approach referring to a sustainability setting will be briefly presented. Thus, we have $i = 1,2, \ldots n$ systems to be evaluated according to $j = 1,2, \ldots m$ sustainability indicators. $R_{ij}$ measures the value reached by the $i$th system when it is evaluated according to the $j$th indicator. $W_j$ represents a preferential weight, that is, the relative importance attached by the Decision Maker (DM) to the $j$th indicator with respect to the others. Also, $p$ is the topologic metric defining the $L_p$ family of distance functions. $K_j$ is a normalizing constant and, finally, $\widehat{R}_j$ represents a desirable level of achievement for the $j$th indicator. $X_i$ is a binary variable, taking the value 1 when the $i$th system is the most sustainable one, otherwise $X_i = 0$, being the value achieved by the objective function the measurement of the sustainability of the $i$th system. Thus, by solving model (1)–(4) $n$ times, incorporating in each iteration the additional constraint $X_k = 1$, when the $k$th system is the most sustainable among the remaining ones, then the ranking of the $n$ systems in terms of sustainability is obtained. The values achieved by the respective objective functions provide the sustainability measurement for each system. See Diaz-Balteiro and Romero (2004a) for a detailed explanation of the whole optimization process. Once all the variables and parameters have been defined, the analytical expression for the MDF model is:

$$Min\ L_p = \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} W_j^p \left| \frac{\left(\widehat{R}_j - R_{ij}\right) X_i}{K_j} \right|^p \right]^{1/p} \tag{1}$$

Underlying the general MDF model given by (1) are an important number of specific models according to the value of the metric $p$ and according to the meaning given to the desirable level of achievement $\widehat{R}_j$. Thus, if $\widehat{R}_j$ is an ideal value, model (1) derives towards a compromise programming approach, if $\widehat{R}_j$ is a "satisficing" target model (2) derives towards a goal programming approach, etc. Regarding the value of the metric, for $p = 1$ model (1) turns into the following one:

$$Min\ L_1 = \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} W_j^p \left| \frac{\left(\widehat{R}_j - R_{ij}\right) X_i}{K_j} \right| \right] \tag{2}$$

Model (2) implies optimizing the average providing the best aggregate performance. However, this solution can give poor results for some of the indicators. This bias can make this type of solution unacceptable from a sustainability perspective. For metric $p = \infty$ model (1) turns into the following one:

$$Min\ L_\infty = D$$

Subject to:

$$\sum_{j=1}^{m} W_j \left| \frac{\left(\widehat{R}_j - R_{ij}\right) X_i}{K_j} \right| - D \le 0 \quad i \in \{1, 2, \ldots n\} \tag{3}$$

$D$ being the maximum deviation. In this way, the "most balanced" solution is obtained. This solution is an appealing one due to its balancing character but it can produce poor "average" results. It is tempting to use metric $p$ as a "balancing factor" between "average" and "balance". However, this type of orientation leads to having to solve complex non-linear and non-convex mathematical programming problems. That is why the above conflict within the MDF approach is usually treated by trading-off $L_1$ and $L_\infty$, with the help of the following model:

$$Min\ L_\lambda = (1 - \lambda) D + \lambda \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} W_j \left| \frac{\left(\widehat{R}_j - R_{ij}\right) X_i}{K_j} \right| \right]$$

Subject to: (4)

$$\sum_{j=1}^{m} W_j \left| \frac{\left(\widehat{R}_j - R_{ij}\right) X_i}{K_j} \right| - D \le 0 \quad i \in \{1, 2, \ldots n\}$$

In model (4) $\lambda$ plays the role of a control parameter. Thus, when $\lambda = 1$ model (4) turns into model (2) and the "average" is optimized and when $\lambda = 0$ , then model (4) turns into model (3) and the "balance" is optimized. For control parameter values belonging to the open interval (0,1) compromise solutions between $L_1$ and $L_\infty$ can be obtained if they exist. One example of these compromise solutions within a sustainability context can be seen in Gómez-Limón and Sanchez-Fernandez (2010). Hence, control parameter $\lambda$ can be interpreted as being a marginal rate of transformation between "average" and "balance". Some clarifications on the meaning and implementation of MDF model in the sustainability field are made in Section 9.

## 3. Indicators and criteria

In talking about criteria and indicators, it is necessary to point out that there is no unanimous consensus in scientific literature on when to use either word or the other. Thus, many works have not differentiated between criteria and indicators (in Diaz-Balteiro et al. (2017a), 25.8% of the articles analyzed did not make that differentiation), and on many occasions they were taken to be synonyms. There is also a frequent hierarchy between the two