Solid-State Electronics 105 (2015) 63–69

Contents lists available at ScienceDirect

Solid-State Electronics

journal homepage: www.elsevier.com/locate/sse

Analytical modeling of cutoff frequency variability reserving correlations due to random dopant fluctuation in nanometer MOSFETs

Wei-feng Lü *, Guang-yi Wang, Ling-ling Sun

Key Laboratory for RF Circuits and Systems of Ministry of Education, Hangzhou Dianzi University, Hangzhou 310018, People's Republic of China

ARTICLE INFO

Article history:
Received 19 July 2014
Received in revised form 15 December 2014
Accepted 18 December 2014
Available online 10 January 2015

The review of this paper was arranged by Prof. A. Zaslavsky

Keywords:
Cutoff frequency
Analytical modeling
Nanometer MOSFET
Random dopant fluctuation

ABSTRACT

Correlations are concerned for modeling of CMOS devices and circuits variability when using back propagation of variations (BPV) methodology in the paper. Strong reverse correlations are observed and investigated between variation parameters, particularly for threshold voltage (V_T), total gate capacitance (C_{gg}) and trans-conductance (g_m), on gate voltage dependence due to random dopant fluctuation (RDF) in nanometer MOSFETs. These correlations are verified both in theoretical and simulation approaches. Based on these correlations, a simple and accurate analytical model for capturing f_T variability is proposed. The model is in good agreement with HSPICE Monte Carlo simulations in different design decisions such as effective width length ratios, source voltages and doping concentrations. Results show the estimation errors are not more than −2.33% and 1.30% for NMOS and PMOS, respectively. Furthermore, our analysis of the correlation and analytical formula are still effective for the continued scaling CMOS technology.

1. Introduction

With the continued scaling of metal oxide semiconductor field effect transistors (MOSFETs) down to nanometer feature size, process variations, which greatly influence CMOS devices and integrated circuits (ICs), play an important role in today's IC design and manufacture [1]. Therefore, analysis and modeling of such variations are critical to the state of art nanometer IC industry and become one of the most popular issues for today's CMOS technology to ensure high performance and yield [2–8]. Most of approaches are based on the well known backward propagation of variation (BPV) methodology since it is easy to characterize circuit performances as well as individual NMOS and PMOS performance parameters. Hence, a lot of researchers have devoted effort to such modeling [9–12]. In addition, process variations for MOSFETs result from many aspects such as manufacturing deviations in effective gate length (L_{eff}), oxide thickness (T_{OX}), and the random dopant fluctuation (RDF) in channel [12,13]. Among these many factors, RDF has become one of the most dominant causes since it seriously impacts both CMOS analog and digital ICs [14,15]. Therefore, various effects associated with RDF have been studied in a very wide range of theoretical and simulation approaches [16,17]. In the design perspective, analytical modeling with BPV methodology that provides insight on how variations affect performance are preferred comparing to the very complex and time consuming Monte Carlo simulations. For this reason, many researchers have donated a lot of efforts on such modeling in this field. The models for MOS current mismatch are presented with different approaches [18,19]. Hamid Mahmoodi et al. propose a semi-analytical model to estimate the RDF- induced effects on CMOS delay by V_T variation [14]. A method is presented by Rahul Rithe for stochastic characterization and computing any point on the probability density function (PDF) of a timing path delay at low voltage [20]. Another paper studies the RDF effect on timing of flip-flops for future technology generations [21]. However, to the best of our knowledge, there is very little work reporting an analytical model and relationship of variation parameters for cutoff frequency (f_T) variability due to RDF in detail. The importance of f_T is because as a fundamental RF figure of merit, it represents the maximal frequency for current amplifying that the MOSFET can generate [22]. So we will address this deficiency. Our derived analytical model is simple and accurate. Moreover, compared to the previous works, the methodology is divergent with the traditional approaches. First, the mathematical basis and the statistical correlation are concerned and emphasized. To the best of our judgment, neglecting correlation in many BPV based applications means simplified formulas at the expense of accuracy. However, this work discovers reserving correlations does not certainly increase model's computational complexity. Second, the relationships between electrical parameters associated with f_T and its variability due to RDF, for the first time, are investigated

* Corresponding author.
E-mail address: lvwf@hdu.edu.cn (W.-f. Lü).

http://dx.doi.org/10.1016/j.sse.2014.12.024
0038-1101/

in detail. The reverse strong correlations are captured between variation parameters in sub-threshold and strong inversion, which gives a complete insight on how RDF influences CMOS RF performance parameters.

The rest of paper is organized as follows. In Section 2, the BPV modeling methodology is reviewed and we emphasize the importance of reserving correlation. Then the correlations between V_T, C_{gg}, and g_m due to RDF are investigated. The proposed analytical model for f_T variability due to RDF is verified with the HSPICE Monte Carlo simulations in Section 3. In addition, the results are analyzed and discussed. We draw the conclusions in Section 4.

2. Modeling approach

2.1. BPV methodology

Let e_i ($i = 1, 2,\ldots,m$) be electrical parameters and p_j ($j = 1, 2,\ldots,n$) be variation parameters for MOSFETs. $\overline{P} = (\overline{p_1}, \overline{p_2}, \ldots, \overline{p_j})$ is the mean of $P = (p_1, p_2, \ldots, p_j)$ and based on the Taylor series expansion of e_i in the neighborhood of $\overline{P} = (\overline{p_1}, \overline{p_2}, \ldots, \overline{p_j})$, we have [10,23]:

$$\Delta(e_i) \approx e_i - e_i(\overline{P}) = \sum_{j=1}^{n} S_{i,j}(p_j - \overline{p_j}) = \sum_{j=1}^{n} S_{i,j}\Delta p_j \quad (1)$$

$$\sigma^2(e_i) \approx \sum_{j=1}^{n} S_{i,j}^2 \sigma^2(p_j) \quad (2)$$

where $S_{i,j} = \frac{\partial e_i(P)}{\partial p_j}\Big|_{P=\overline{P}}$ is the sensitivity, $\sigma^2(e_i)$ and $\sigma^2(p_j)$ are the variances for e_i and p_j, respectively. The above well known BPV methodology has been applied in a wide range for statistical modeling [9–11,14,18]. However, an important assumption that underlie (2) has to be noted: the variation parameters are mutually independent; otherwise we must correct it as

$$\sigma^2(e_i) = \sum_{j=1}^{n} S_{i,j}^2 \sigma^2(p_j) - 2\sum_{j=1}^{n}\sum_{k=j+1}^{n} \gamma_{j,k} S_{i,j} S_{i,k} \sigma(p_j)\sigma(p_k) \quad (3)$$

where $\gamma_{j,k}$ is the correlation coefficient between p_j and p_k. It seems considering correlations between variation parameters will greatly increase the complexity of computing. That is why $\gamma_{j,k}$ are usually regarded as zero if no clear correlations are found [11]. Certainly, such kind of processing will reduce the accuracy of variability modeling. Let us assume $\sigma(p_j)$ are equal and all the correlation coefficients between p_j and p_k are equal to -1. Then we have [23]

$$\sigma^2(e_i) = n\sigma^2(p_j) + 2\frac{n(n-1)}{2}\sigma^2(p_j) = n^2\sigma^2(p_j) \quad (4)$$

This is completely different from the result supposing mutual independence between p_j and p_k; that is $\sigma^2(e_i) = n\sigma^2(p_j)$. Therefore, it is indispensable to investigate the correlations between variations parameters for modeling of MOS integrated devices and circuits.

2.2. Variability modeling

The MOSFETs investigated are mainly with 45 nm process technology. However, the analyses of correlation and modeling approach are also suitable for the more advanced CMOS technology node as we can see below. The effective channel dimensions ($L_{eff} \times W_{eff}$) are 22.5 nm × 40 nm and 22.5 nm × 80 nm for NMOS and PMOS, respectively, and the nominal channel doping concentrations (N_{av}) are 3.24×10^{18} cm^{-3} and 2.44×10^{18} cm^{-3}, respectively, based on PTM model [24]. The standard deviation of dopant atoms in MOSFET channel ($\sigma[N]/\mu[N]$) is inversely proportional to the root of N according to the Poisson statistical distribution, where N is the number of dopant atom in channel. We have the mean of N $\mu(N) = N_{av}v_B$, where v_B is the multiplication of depletion and channel area $v_B = W_{eff}L_{eff}x_d$ and x_d is the depletion layer thickness $x_d = \sqrt{\frac{4\varepsilon_0\varepsilon_{si}|\Phi_F|}{qN_{av}}}$ [18,25,26]. In the interpretation of modeling procedure, it is useful to rely on simple formula. The basic assumption of f_T is the validity of the low-frequency approximation to the parameters and the absence of gate leakage [27]. Then f_T for a MOSFET can be described in a well known analytical formula [11,22]

$$f_T = \frac{g_m}{2\pi C_{gg}} \quad (5)$$

where g_m and C_{gg} are trans-conductance and total gate capacitance of the MOSFET. As we know, g_m is described as the sensitivity of MOS drain current with gate voltage. Therefore, parameter variations in both g_m and C_{gg} have to be taken into account when estimating f_T variability. It is not difficult to understand that f_T changes with bias conditions since not only g_m but also C_{gg} are gate voltage dependent and these dependences are depicted in Fig. 1 with RDF effect.

If no clear correlations are observed between g_m and C_{gg}, or they are regarded as mutually independent, based on BPV methodology, we have [11]

$$\frac{\sigma^2(f_T)}{f_T^2} = \frac{\sigma^2(g_m)}{g_m^2} + \frac{\sigma^2(C_{gg})}{C_{gg}^2} \quad (6)$$

From (6), we can easily conclude that f_T variability ($\sigma(f_T)/f_T$) is more than g_m and C_{gg} variations but between the addition and subtraction of g_m and C_{gg} variations. However, our HSPICE Monte-Carlo simulations contradict with above analysis and the clear difference between the Eq. (6) and simulation data is depicted in Fig. 2. The estimation error of this model is average 98.8%. So the traditional BPV-based model cannot describe f_T variability due to RDF correctly though it is effective for the uncertain fluctuation sources in CMOS manufacturing [11]. The reason is that the existence of correlation between g_m and C_{gg} if only the RDF effect is considered. Thus, the assumption of independence of them will lead to an unacceptable error for f_T variability estimation [28].

The relationships of g_m and C_{gg} with doping concentrations (N_{av}) are investigated both in theoretical and simulation approaches. For simplicity, the NMOS operation region is separated into sub-threshold and strong inversion according to the gate bias voltage (V_{gs}) and the minimum C_{gg} (C_{ggmin} at $V_{gs} = V_T$). Then g_m can be expressed approximately as [28–30]

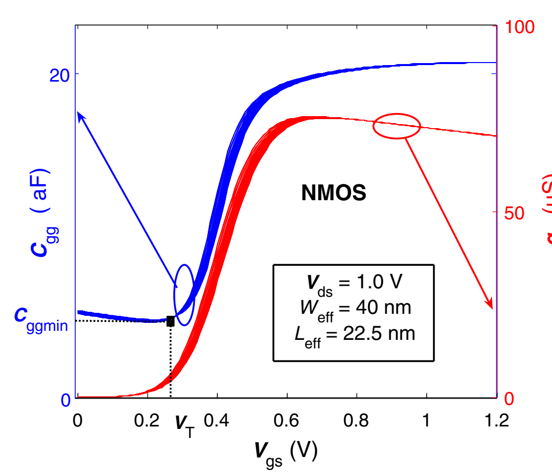

Fig. 1. Variations of g_m and C_{gg} with V_{gs} dependence.