Research analysis and derivation of copula-based drought Severity-Area-Frequency curve in Lake Urmia basin, Iran

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Abstract
Droughts are extreme events characterized by temporal duration and spatial large-scale effects. In general, regional droughts are affected by general circulation of the atmosphere (at large-scale) and regional natural factors, including the topography, natural lakes, the position relative to the center and the path of the ocean currents (at small-scale), and they don’t cover the exact same effects in a wide area. Therefore, drought Severity-Area-Frequency (S-A-F) curve investigation is an essential task to develop decision making rule for regional drought management. This study developed the copula-based joint probability distribution of drought severity and percent of area under drought across the Lake Urmia basin, Iran. To do this end, one-month Standardized Precipitation Index (SPI) values during the 1971–2013 were applied across 24 rainfall stations in the study area. Then, seven copula functions of various families, including Clayton, Gumbel, Frank, Joe, Galambos, Plackett and Normal copulas, were used to model the joint probability distribution of drought severity and drought area. Using AIC, BIC and RMSE criteria, the Frank copula was selected as the most appropriate copula in order to develop the joint probability distribution of severity-percent of area under drought across the study area. Based on the Frank copula, the drought S-A-F curve for the study area was derived. The results indicated that severe/extreme drought and non-drought (wet) behaviors have affected the majority of study areas (Lake Urmia basin). However, the area covered by the specific semi-drought effects is limited and has been subject to significant variations.

1. Introduction
Drought is a long-term and natural phenomenon that affects large areas and it is characterized by rainfall events less than expected rainfall. Compared to other natural disasters such as floods and storms, droughts are geographically more comprehensive and widespread (Obasi, 1994). Once the drought exceeds a prolonged season or period, it leads to water shortages and the insufficient water resources permeates to social, economic and environmental damages (WMO, 2006). Since temporal and spatial characteristics of drought are considerably different from one region to another, they are considered as multivariate phenomena characterized by such variables as duration, severity, intensity and area under drought (areal extent). Although at-site drought analysis can provide useful local information, it seems that these results will be embedded with high levels of uncertainty in drought management or drought risk assessment in a wide area (Tallaksen and Hisdal, 1997; Hisdal et al., 2001). Usually, natural short/long-term droughts occur regionally and they often affect very large area. Thus, if droughts are studied in the context of regional analysis, it will be feasible to conduct more comprehensive assessment on this phenomenon in any given region. In fact this type of analysis is considered a crucial requirement of short-term and long-term strategic planning in water resources management (Mishra and Singh, 2011). Given such an analysis, drought duration and severity as well as the percent of area affected by drought should be evaluated thereof (Tase, 1976; Sen, 1980, 1998; Santos, 1983; Rossi et al., 1992; Vogt and Somma, 2000; Bonaccorso et al., 2015).

Several studies have been conducted in order to understand the nature of spatial and temporal characteristics of droughts in the regional scale. The regional drought risk can be gauged through multivariate relationships, including drought Severity-Area-Frequency curves (S-A-F) or drought Intensity-Area-Frequency curve (I-A-F) (Reddy and Ganguli, 2013). These curves are

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calculated on the basis of mathematical relationships between drought severity or intensity and its coverage area for different return periods (Alemaw and Kileshye-Onema, 2014; Khadr, 2016). As such, extreme value probability distributions are usually used to extract such curves. To do this, the severity of the drought is extracted from the concerned historical event in order to determine the percent of area under drought and, then, the frequency analysis is done accordingly.

Shin and Salas (2000) proposed a new method to analyze and quantify the spatial and temporal patterns of drought on the basis of annual rainfall data and neural networks in southwestern region of Colorado, USA. Kim et al. (2002) analyzed the spatial and temporal characteristics of drought using Palmer Drought Severity Index (PDSI) with developing the Intensity-Area-Frequency curve in Conchos River basin, Mexico. In this case, they made use of Kriging technique to interpolate the values of drought index in the region. Having derived the Severity-Area-Frequency curve, Hssal and Tallaksen (2003) attempted to model the monthly rainfall and streamflow time series in Denmark using the Empirical Orthogonal Functions (EOF). Loukas and Vasiladiades (2004) gauged the temporal and spatial variability of meteorological droughts from 1960 to 1993 in Thessaly, Greece. They resorted to monthly rainfall data over 50 meteorological stations as well as multiple regression method in order to determine the area under drought. Then, they developed the Severity-Area-Frequency curve using SPI across multiple time scales in the study area. Mishra and Desai (2005) determine the Severity-Area-Frequency relations using SPI for the period 1965—2001 in Kansabati River basin, India. They made use of Inverse Distance Weighting (IDW) method to interpolate the values of SPI. Santos et al. (2010) examined the meteorological drought in Portugal in the period 1910—2004 using SPI with 1-, 6- and 12-month time scales. Finally, concluded that the southern part of the country endured more severe drought impacts than the northern part of the country in this regard.

The forenamed studies indicate that different methods have been used to analyze two variables of drought severity and area under drought. However, these methods are embedded with serious limitations, such as limiting the analysis based on historical data, the impossibility of extracting the predictions of long-term drought events, failure to maintain a full correlation between two dependent variables and identical marginal probability distribution of dependent variables.

In recent years, the concept of copula functions has been widely used as a new advanced technique to model the bivariate or multivariate joint probability distributions in hydrology and water resources engineering (De Michele and Salvadori, 2003; Zhang et al., 2012; Ganguli, 2014; Salvadori and De Michele, 2015; Moazami et al., 2014; Tosunoglu and Can, 2016; Liu et al., 2016). The copula functions are a flexible and highly efficient technique to model the characteristics of two or more dependent variables of natural phenomena such as drought duration and severity or severity and area under drought. This is due to the fact that the copula functions maintain a full correlation between such variables and that they are not limited to identical marginal probability distribution of dependent variables in terms of long-term prediction of drought events (Zhang and Singh, 2006).

Shiau (2006) examined the bivariate joint probability distribution of drought duration and severity in the southern part of Taiwan using the SPI and theory of copula. Song and Singh (2010) modeled the joint probability distribution of duration, severity and inter-arrival time using the trivariate Plackett copula in China. Reddy and Ganguli (2012) applied four different families of bivariate copula, including Archimedean, Extreme values, Plackett and Elliptical families, to model the joint probability distribution of drought characteristics and, consequently, concluded that Gumbel—Hougaard copula was the best and most appropriate copula. Finally, they used Gumbel—Hougaard copula to derive the drought Severity-Duration-Frequency curve in Rajasthan, India. Mirabbasi et al. (2012) made use of bivariate copula to model the drought duration and severity in northwest parts of Iran. Regarding the results of RMSE and tail dependence coefficients, it was concluded that Galambos copula was the best copula in terms of modeling the drought duration and severity. Also, joint probability characteristics and conditional return periods were analyzed in the region. Reddy and Ganguli (2013) employed gridded data and copula function in order to analyze the spatial and temporal characteristics of drought in Kansabati, India, and develop the Intensity-Area-Frequency curve in the region. All these studies stressed the capability of copula technique in modeling bivariate or multivariate dependent drought characteristics. In addition, comprehensive reviews of the application of copula have been reported in the Genest and Favre (2007) and Salvadori et al. (2007).

Accordingly, this study attempted to regionally analyze the drought severity and area under drought in Lake Urmia basin, as one of the most important basin with relatively high water potential in Iran. Currently, the Lake Urmia was faced with a sharp and critical decline in the water level. Besides, it was noted that this was the first research that regionally analyzed the temporal and spatial variations in drought events emphasizing on dependency of drought severity and area under drought as a bivariate analysis in the study area.

The main objectives of this study were as follow: 1) Modeling drought characteristics in the Lake Urmia basin, Iran, 2) Evaluating the performance of different families of copula in modeling the structure of drought characteristics and 3) Deriving the drought Severity-Area-Frequency curve on the basis of the selected appropriate copula in the study area.

2. Material and methods

2.1. Drought characteristics

In this study, the SPI index was used for drought monitoring and the derivation of the one-month SPI values. The SPI is the most appropriate and suitable meteorological drought index in reducing the uncertainty embedded with selecting the inappropriate drought index in analyzing the joint probabilities of droughts (Esfahanian et al., 2017; Montaseri and Amirataee, 2017; Montaseri et al., 2017). The values of any given SPI for each period and at any given station might be calculated as follows: Firstly, the rainfall time series are transferred from the own distribution to the standard normal distribution. Then, the values of SPI are estimated using $SPI_t = (P_t - \overline{P})/\sigma$, where, $SPI_t$ and $P_t$ represent the drought index and normalized rainfall values in period $t$, respectively, and $\overline{P}$ and $\sigma$ are the mean and standard deviation of normalized rainfall time series. Finally, the drought periods are determined in relation to value of SPI located below a specified threshold (McKee et al., 1993).

In specific, the drought characteristics used in this study were as follows: 1) The percent of area under drought ($A_t$): it was equal to a specified percent of total area in period $t$ in which the value of SPI was below a specified threshold (Reddy and Ganguli, 2013). Accordingly, the percent of area under drought ($A_t$) for period $t$ was calculated by Equation (1).

$$A_t = \frac{\sum_{i=1}^{N_y} I [SPI_{it} \leq SPI_{thr}]}{\sum_{i=1}^{N_y} a_i}$$ for $t = 1, T$ (1)

where, $I[.]$ is the logical indicator function taking the value 1 if its