



## Research article

# A fractional-factorial probabilistic-possibilistic optimization framework for planning water resources management systems with multi-level parametric interactions

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## ABSTRACT

In this study, a multi-level factorial-vertex fuzzy-stochastic programming (MFFP) approach is developed for optimization of water resources systems under probabilistic and possibilistic uncertainties. MFFP is capable of tackling fuzzy parameters at various combinations of  $\alpha$ -cut levels, reflecting distinct attitudes of decision makers towards fuzzy parameters in the fuzzy discretization process based on the  $\alpha$ -cut concept. The potential interactions among fuzzy parameters can be explored through a multi-level factorial analysis. A water resources management problem with fuzzy and random features is used to demonstrate the applicability of the proposed methodology. The results indicate that useful solutions can be obtained for the optimal allocation of water resources under fuzziness and randomness. They can help decision makers to identify desired water allocation schemes with maximized total net benefits. A variety of decision alternatives can also be generated under different scenarios of water management policies. The findings from the factorial experiment reveal the interactions among design factors (fuzzy parameters) and their curvature effects on the total net benefit, which are helpful in uncovering the valuable information hidden beneath the parameter interactions affecting system performance. A comparison between MFFP and the vertex method is also conducted to demonstrate the merits of the proposed methodology.

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## 1. Introduction

Global water resources are under pressure due to rapid population growth, intensive socio-economic development, and warming climate (Vörösmarty et al., 2000). Water scarcity is becoming a critical issue in many countries, since water demand is increasing rapidly while fresh water supplies are shrinking dramatically. As a result, conflicts often arise when different water users compete for a limited water supply (Wang et al., 2008). To achieve sustainable development, wise decisions are desired to make best use of limited water resources.

Optimization techniques are recognized as a powerful tool for investigating the economic benefits of policy decisions and for

planning water resources systems in an effective and efficient way. However, mathematical modeling of real-world water resources systems involves a variety of uncertainties due to (1) the inherent unpredictability of systems, (2) simplifications in model formulation, and (3) uncertainties in the estimates of model parameters. The potential interactions among these uncertainties may further intensify the complexity in the decision process. Therefore, inexact optimization methods are desired to support water resources management in an uncertain and complex environment.

Over the past decades, a number of inexact optimization methods have been proposed for addressing uncertainties in environmental management problems (Maqsood et al., 2005; Chung et al., 2009; Teegavarapu, 2010; Gaivoronski et al., 2012; He et al., 2012; Shen et al., 2012; Wang et al., 2012; Assumaning and Chang, 2014; Li et al., 2015; Rahmani and Zarghami, 2015). Among these methods, two-stage stochastic programming (TSP) has the ability to take corrective actions after a random event occurs (Birge and Louveaux, 1988). In a TSP model, two groups of

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decision variables can be distinguished. The first-stage decision must be made before the values of uncertain parameters are observed. The optimal values of the first-stage decision variables are independent of the realization of uncertain parameters. Subsequently, the second-stage decision can be determined after a specific realization of uncertain parameters is observed. The recourse action in the second stage is effective in minimizing the risk of infeasibility as a result of the first-stage decision. TSP can thus be used to make decisions in a two-stage fashion with uncertain parameters expressed as probability distributions. However, TSP has the difficulty in dealing with uncertainties when the sample size is too small to generate distribution functions. The large sample size required for constructing the parameters probability distributions leads to a major limitation to the practical applicability of TSP.

In comparison, fuzzy set theory, which serves as a useful mathematical tool to facilitate the description of complex and ill-defined systems (Zadeh, 1965, 1978), is capable of quantifying the vague information without the sample size requirement. Fuzzy mathematical programming based on fuzzy set theory has been widely studied (Bellman and Zadeh, 1970; Zimmermann, 1976; Tanaka and Asai, 1984; Zimmermann, 1985; Dong and Shah, 1987; Teegavarapu and Simonovic, 1999; Akter and Simonovic, 2005; Jiménez et al., 2007; Wang and Huang, 2013; Wang et al., 2015b). Among them, the vertex method proposed by Dong and Shah (1987) is recognized as an effective tool for tackling fuzzy sets based on the  $\alpha$ -cut concept and interval analysis. Nevertheless, the vertex method assumes that the same  $\alpha$ -cut level is applied to all fuzzy sets in the fuzzy discretization process, and the results can thus be obtained with a particular  $\alpha$ -cut level. In real-world problems, however, decision makers may have different attitudes towards fuzzy parameters, implying that the  $\alpha$ -cut levels applied to different fuzzy parameters may vary. The  $\alpha$ -cut levels are chosen subjectively based on decision makers' attitudes towards uncertainty. The larger the specified value of  $\alpha$ , the smaller the uncertainty. For example, certain fuzzy parameters may be tackled with the  $\alpha$ -cut level of 0.5, while the others are processed with the  $\alpha$ -cut level of 1.0. Such a complexity needs to be addressed when dealing with fuzzy parameters. In addition, fuzzy parameters are often correlated with each other in practice. It is thus necessary to investigate the potential interactions among fuzzy parameters and reveal their effects on the model response.

Factorial designs have been widely used to study the interaction effects of two or more factors on a response variable (Lewis and Dean, 2001; Lin et al., 2008; Qin et al., 2008; Mabilia et al., 2010; Onsekizoglu et al., 2010; Zhang and Huang, 2011; Wei et al., 2013; Wang et al., 2015a). All these studies took advantage of the two-level factorial design which assumed that the response was linear over the range of factor levels. However, many practical problems involve the nonlinear relationships between design factors and the model response. The two-level factorial experiment can hardly address the nonlinear effects. The concept of multi-level factorial designs is thus proposed in this study to detect the curvature in the response function (Box and Behnken, 1960; Xu et al., 2004; Wu and Hamada, 2009; Wang and Huang, 2015). When the investigated factors are uncertain and given as fuzzy sets instead of the standard representation of design factors with each at fixed levels, combining the multi-level factorial designs with the vertex method is a sound strategy for not only revealing the interactions among fuzzy sets, but also facilitating the processing of fuzzy sets using various combinations of  $\alpha$ -cut levels.

Therefore, the objective of this study is to develop a multi-level factorial-vertex fuzzy-stochastic programming (MFFP) approach through incorporating TSP, fuzzy set theory, vertex analysis, and the concept of multi-level factorial designs within a general

framework. MFFP is capable of tackling probabilistic and fuzzy uncertainties, and of revealing the interdependences of fuzzy uncertainties as well as their resulting effects on system performance. A water resources management problem will be used to verify the applicability of MFFP. Finally, a detailed comparison between MFFP and the vertex method will be conducted to demonstrate the merits of the proposed methodology.

## 2. Methodology

### 2.1. Fuzzy stochastic optimization model

In a TSP model, decision variables can be divided into two groups, including first-stage and second-stage variables. First-stage variables are decided upon prior to the actual realization of random parameters. Once the uncertain events have occurred, a recourse action can be taken in the second stage, leading to a dynamic decision-making process. A standard TSP model can be formulated as follows (Birge and Louveaux, 1988):

$$\text{Max } f = C^T X + E[Q(X, \xi)] \quad (1a)$$

subject to:

$$AX \leq B \quad (1b)$$

$$X \geq 0 \quad (1c)$$

With

$$Q(X, \xi) = \text{max} D(\xi)^T Y \quad (1d)$$

subject to:

$$T(\xi)X + W(\xi)Y \leq H(\xi) \quad (1e)$$

$$Y \geq 0 \quad (1f)$$

where  $C \in R^{n_1}$ ,  $X \in R^{n_1}$  (first-stage decision variable vector),  $A \in R^{m_1 \times n_1}$ ,  $B \in R^{m_1}$ ,  $D \in R^{n_2}$ ,  $Y \in R^{n_2}$  (second-stage decision variable vector),  $T \in R^{m_2 \times n_1}$ ,  $W \in R^{m_2 \times n_2}$ ,  $H \in R^{m_2}$ ,  $\xi$  is a random vector and  $\xi$  ( $D, T, W, H$ ) contains the data of the second-stage problem. Letting the random vector  $\xi$  take a finite number of possible realizations  $\xi_1, \dots, \xi_k$  with respective probability of occurrence  $p_1, \dots, p_k$ ,  $\sum p_k = 1$ , the above TSP problem can be written as a deterministic equivalent linear program as follows:

$$\text{Max } f = C^T X + \sum_{k=1}^m p_k D^T Y \quad (2a)$$

subject to:

$$AX \leq B \quad (2b)$$

$$TX + WY \leq \xi_k, \quad \forall k \quad (2c)$$

$$X \geq 0 \quad (2d)$$

$$Y \geq 0 \quad (2e)$$

TSP is capable of reflecting the dynamic nature of decision problems under uncertainty, enhancing the flexibility in the decision-making process. In TSP, random variables in the model's parameters can be represented by a set of scenarios, each occurring with a given probability. In real-world problems, subjective

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