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Shot noise in resistively coupled single tunnel junctions

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ABSTRACT

This paper presents a Monte-Carlo method based on the distribution of the time between successive tunnel events in resistively coupled nanoscale tunnel junctions. The frequency dependent Fano factor is computed for this structure and it is shown that the zero-frequency factor decreases with increasing coupling resistance. Studying the dependence of the Fano factor on the applied voltage has revealed an optimum bias value at $\sim 2.5e/C$–$5e/C$. This technique could be further developed to investigate complex single–electron tunnelling (SET) structures with resistive elements.

1. Introduction

The phenomena based on the Coulomb blockade (CB) and Single-electron tunnelling (SET) have attracted a lot of attention during the last two decades [1]. Furthermore, devices based on the CB and SET hold a promise of augmenting the current VLSI technologies and present an excellent platform for advancing our understanding of the physics of low dimensional nanoscale structures. The SET effects have been observed and studied in semiconductor nanostructures, metallic quantum dots and low-dimensional organic nanostructures. SET circuits are prone to undesirable effects caused by thermal and shot noise [2]. Furthermore, $1/f$ noise has been reported in experimental results and has been attributed to the trapping and de-trapping of discrete charges in the vicinity of the junctions resulting in a shift of the threshold voltages [3]. Effects of thermal fluctuations on resistively couple single electron transistor have shown that the CB could be smeared out by the thermal noise introduced by the coupling environment [3].

Shot noise is a consequence of the discrete nature of the charge passing through the junctions. In case of uncorrelated current, the shot noise is Poissonian and is characterised by a power spectral density function 2 eI. Non Poissonian noise in SET structures has been studied both experimentally [4] and theoretically [5]. Most of the structures reported in literature include two and three junction systems together with long arrays of tunnel junctions, or derivatives of such structures. Analysis techniques used in investigating the coherence of signals in SET systems include quantum mechanical models, full counting statistics techniques [6] and semi-classical models [7]. It is pointed out that the quantum fluctuations smear out the CB in tunnel junctions. The tunnel junction is normally connected to the source and to other components in the circuit via a coupling circuit that is has resistive and reactive properties. The electromagnetic properties of the coupling circuit could be modelled as a standard transmission line. In [8] the effect of the quantum fluctuations were studied by investigating the modes generated in the electromagnetic environment as a result of the tunnel event. It is also shown that the tunnel event is affected by the length of the coupling transmission line that is covered by a travelling wave during the uncertainty length. The horizon model predicts that the stray capacitance influencing a tunnel event increases linearly with the uncertainty time, τ_v. This implies the stray capacitance, and hence the threshold voltage of tunnelling would change with the voltage across the tunnel junction. The horizon length is computed as $r = \tau_v v_{ph}$ where v_{ph} is phase velocity of propagation of the electromagnetic wave in the transmission line. The environment impedance within this radius should be taken into consideration in any realistic model. The effects will be more pronounced at low voltages.

Following a tunnel event, the capacitor will have to be recharged to the threshold level before another event is possible again. The external resistance helps to control the classical charging process and therefore introducing a correlation between successive tunnel events.

Metallic arrays with on-chip chromium micro-strip resistors are reported to have a total resistance of ~ 60 kΩ, [9], and resistances as high as 0.4 MΩ have been achieved [10]. This ohmic resistance

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is higher than the resistance quantum and therefore causes enough damping to achieve classical electron dynamics. The increased coupling resistance has the additional effect of reducing the cotunnelling events in multi-junction systems, e.g. [11].

Shot noise in a single junction connected to a source via a resistive path could be studied using the orthodox theory of tunnelling. The steady-state solution of the Fokker–Plank master equation would provide the charge distribution; which could then be used to compute the properties of noise and other steady-state properties see [1]. In [12–14], noise properties were studied using the distribution of time between tunnel events to compute the ratio of the standard deviation of the time between successive tunnel events to the mean value.

It is known that a resistive element in series with a nanoscale single tunnel junction is a necessary requirement to achieve and observe time correlated tunnelling events. The same effect of quasi-continuous transfer of charge is achieved in long arrays of tunnel junctions that are strongly coupled to the voltage source.

In recent years, the Fano factor has been accepted as a reliable indicator of the effect of noise on the performance of SET devices. This factor reflects the performance of the circuit under study in relation to a reference circuit carrying the same average dc current where the reference circuit exhibit Poissonian transport properties. Ref. [14] gives an expression of the Fano factor for a single junction circuit with an external resistance. This model provides an approximation of the power spectral density around the resonant frequency and predicts a sub-Possionian Fano factor (<1) that vanishes in the infinite series resistance regime. It is pointed out that the model of [14] is accurate only around the power spectral peaks, while the accuracy rapidly deteriorates away from the peaks.

In this paper, we present a hybrid technique based on the distribution of time between successive tunnel events and the Monte-Carlo method to compute the Fano factor. The results based on this technique are compared against analytical results.

The rest of the paper presents a theoretical background, describes the method used in the noise calculation and presents results extracted from the Monte-Carlo calculations.

2. Theory

The coupling electromagnetic environment may be modelled using a distributed transmission line model, including capacitive, inductive and resistive elements. The leakage conductance between the micro-strip and the ground plane is assumed to be negligibly small. The full transmission line model could be simplified by using an equivalent π-model, with lumped elements or a lumped T-equivalent circuit.

In this paper, the properties of SET oscillations in a single junction are studied by investigating the distribution of time between successive tunnel events. At a fixed voltage across the junction, the temporal behaviour of tunnel events satisfies a Poisson process where the time between events is characterised by a negative exponential distribution. If the instantaneous rate of tunnelling is Γ, then the probability that a tunnel event takes place during a small time interval, dt, is given as Γdt.

Referring to Fig. 1, let $q(t)$ be the value of the electric charge on the capacitor after a time t. We consider the system at temperature $T = 0$ K and note that tunnelling is completely blocked when $q < e/2$ due to the Coulomb effects. Assuming that our observation has started at time $t = 0$, when the charge on the junction is found to be $q(0) = q_i$, the probability density function of an a tunnel event taking place after a time τ can be expressed as:

$$g(\tau) = \Gamma(q(\tau)) \exp\left(-\int_0^{\tau} \Gamma(q(t))dt\right) \tag{1}$$

The evolution with time of the charge across the tunnel junction is shown schematically in Fig. 1. If the external resistance, R_s, is much higher than the quantum resistance, $R_s > h/e^2$, then the charge variable is considered a continuous accumulation of charge on the junction. Once the charge becomes $q \geqslant e/2$, it will be energetically possible for an electron to tunnel from one side of the junction to another, reducing the charge on the junction by e. If $R_t \ll R_s$, then the tunnel event will take place at or just above the $q = e/2$ threshold level. It is then useful to study the distribution of the time spent above the Coulomb threshold, $q = e/2$, before an electron manages to tunnel through the barrier.

Let the initial condition $q(0)=q_i = e/2$, then the classical evolution of the charge variable on the junction is expressed as a function of time as:

$$q(\tau) = (q_i - CV)\exp\left(-\frac{\tau}{R_s C}\right) + CV \tag{2}$$

For shot noise studies, we consider an environmental condition $T \sim 0$ K,$k_B T \ll e^2/2C$. The tunnelling rate is expressed as:

$$\Gamma(q) = \frac{1}{eCR_t}\left(q - \frac{e}{2}\right) \tag{3}$$

Substituting (3) in (1), the distribution function of the time spent above the Coulomb threshold is found as [12]:

$$g(\tau) = \left(\frac{q(\tau) - e/2}{eR_t C}\right)\exp\left[\frac{R_s}{eR_t}(CV - q_i)\left(1 - \exp\left(-\frac{\tau}{R_s C}\right)\right) - \frac{\tau}{eR_t C}(CV - e/2)\right] \tag{4}$$

As a result of a tunnel event, the charge on the junction drops to $q(\tau)-e$. Let the time it takes the source to recharge the capacitor to the level from $q(\tau)-e$ to $e/2$ be equal to τ_r. The time τ_r is found to be:

$$\tau_r(\tau) = R_s C \cdot \log\left(\frac{CV + e - q(\tau)}{CV - e/2}\right) \tag{5}$$

Let the next tunnel event take place during a small time interval around τ_2 above $e/2$. The time between these successive tunnel events is then given by the following relation:

$$t(\tau) = \tau_r(\tau) + \tau_2 \tag{6}$$

It is assumed that the traversal time for tunnelling and the thermal relaxation time inside the electrodes are much shorter than the time between successive tunnelling events. For $V < 1.5e/C$, the variable τ is an independent random variable. The probability that the system charges above $e/2$ for a time τ followed by a time τ_2 above the $e/2$ threshold resulting in a time between successive tunnel events = t is given by:

$$r(t)|_{\tau,\tau_2} = g(\tau)d\tau \cdot g(\tau_2)d\tau_2 \tag{7}$$

The marginal distribution function of the time between successive tunnel events, $f(t)$, is evaluated by summing the contributions of all possible events characterised by τ and τ_2 that satisfy a total time between successive events of t. Thus, the distribution is given as:

$$f(t) = \int_{\tau} g(\tau)g(t - \tau)d\tau, \quad \text{for } V < 1.5e/C \tag{8}$$

If $CV < 3e/2$ then any tunnel event will leave the junction at a state with $-0.5e < q < 0.5e$, i.e. within the CB region. On the other hand, if $CV > 1.5e$ then any tunnel event occurring at $q > 1.5e$ will leave the junction with $q > e/2$ and an immediate tunnel event is