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Error analysis of theoretical model of angular velocity sensor based on magnetohydrodynamics at low frequency

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ABSTRACT

The angular velocity sensor is commonly used in the navigation of aircrafts, ships and satellites, which can accurately determine the orientation of the moving objects. The angular velocity sensor based on magnetohydrodynamics (MHD) has a wide bandwidth, long life, impact resistance and other excellent properties. However, this sensor has poor performance at low frequency, and there are errors in the theoretical model. This paper presents the derivation of the simplified model by MHD governing equations and analyzes the three-dimensional unsteady motion of conducting fluid in the cylindrical container with a closed rectangular flow channel by the finite volume method. And then simulations and experiments are conducted in the amplitude-frequency and phase-frequency characteristics test, calibration experiment and slope test. Comparing the simplified model, the simulations and experiments, the conclusion can be drawn that the efforts of the secondary flow and the inhomogeneity of the magnetic field cannot be ignored; magnetic field generated by the induced current is almost negligible. The research results of this paper will provide a reference for improving low frequency performance of the sensor.

1. Introduction

The angular velocity sensors are applied to detect the inertial angular motion of the carrier in the air, sea and space [1]. They are the most critical components in the inertial navigation system (INS) [2,3], and their characters largely determine the performance of the INS. Whether the study of traditional gyroscopes or other angular velocity sensors based on new principles pursue higher precision, longer lifetime and greater bandwidth [4,5], a new angular velocity sensor based on magnetohydrodynamics (MHD) [6–8] has been developed. This sensor [9,10] has no mechanically moving parts and requires no power consumption except for amplification circuits. Specifically, it can be used in acceleration environments.

Iwata [12], Pittelkau [11] and Kang [13] discuss the application in three aspects: jitter measurement, attitude determination and head kinematics, respectively. Martin et al. [14] verify that the angular acceleration in an impact environment can be obtained by differentiating a MHD sensor that is investigated by the University of Virginia. However, this method has low resolution and poor consistency because of the inherent error of the sensor. Merkle et al. [15] indicate that noise levels pose a significant problem when determining the angular displacement by a MHD angular velocity sensor for a pendulum test. Pinney et al. [16] show that the error of the sensor is not constant over the measurement interval and they measure the noise and drift characteristics of a MHD sensor by the coherence function. Laughlin et al. [7,17] present a MHD angular velocity sensor developed by Applied Technology Associates and suggest the simplified model, the frequency response function and the angular displacement noise. At present, literature about angular velocity sensors is confined to summary, basic principles, applications and performance testing [7,13,15,17,18], but there is no explanation about why the errors are generated. The existing simplified model ignores some items including the secondary flow, and the uniformity of the magnetic field. And there are principle errors in the theoretical model. In order to ultimately reduce and even eliminate the effects of drift and other errors on navigation accuracy, the errors need to be analyzed and the theoretical model should be corrected and compensated.

This paper is organized as follows. Section 2 introduces brief working principles of the sensor, the MHD governing equations and the derivation of the simplified model. Section 3 presents the iterative model through an analysis of the unsteady flow process of viscous incompressible conducting fluid in an annular closed fluid channel with rectangular cross section by the finite volume method. Sections 4 and 5 discuss the simulations and experiments comprising the amplitude-frequency and phase-frequency

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characteristics test, calibration experiment and slope test and further propose the errors of the simplified model at low frequency. Section 6 concludes this paper.

2. Theory

This section is to describe the brief working principles of the MHD angular velocity sensor and deduce the simplified model starting from the governing equations.

2.1. Brief working principle of the sensor

In Fig. 1, when the hollow cylindrical container starts to rotate around the central axis, the fluid remains stationary due to the effect of inertia. The phenomenon occurs that fluid cuts the magnetic induction lines. And then the motional electromotive force (emf) emerges therefrom, which inhibits the movement of the fluid in turn. This emf reflects the value of angular velocity. Angular acceleration can be calculated by differentiation, and angular displacement can be obtained by integrating given the initial values of angle and angular velocity.

2.2. The governing equations

Assuming the electric conductivity and density of the incompressible fluid are constant, the governing equations of the MHD principles [19,20] can be expressed as:

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right]=\rho\boldsymbol{g}-\nabla p^{*}+\frac{1}{\mu}(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}+\mu_f\nabla^2\boldsymbol{v}, \tag{2.1}$$

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B})+\lambda\nabla^2\boldsymbol{B}, \tag{2.2}$$

where $\boldsymbol{v}$ is the fluid velocity vector, $\boldsymbol{B}$ is the magnetic induction vector, ρ is the density, $\boldsymbol{g}$ is the gravity, μ is the magnetic permeability, $\eta_f=\mu_f/\rho$ is the kinematic viscosity coefficient, μ_f is the dynamic viscosity coefficient, $p^*=p+B^2/2\mu$ is the total pressure, σ is an electrical conductivity, and $\lambda=1/\mu\sigma$ is the magnetic diffusivity. This paper is to discuss the rotating motion of conducting fluid in a hollow cylindrical container, and thus the MHD governing equations should be established in cylindrical coordinates. With $\boldsymbol{v}=(v_r, v_\theta, v_z)$ and $\boldsymbol{B}=(B_r, B_\theta, B_z)$ in (r, θ, z), the above two equations can be given by:

In addition, the mass conservation can be recast as:

$$v_r+r\frac{\partial v_r}{\partial r}+\frac{\partial v_\theta}{\partial \theta}+r\frac{\partial v_z}{\partial z}=0. \tag{2.5}$$

Since the magnetic field is the solenoidal vector field, there is

$$B_r+r\frac{\partial B_r}{\partial r}+\frac{\partial B_\theta}{\partial \theta}+r\frac{\partial B_z}{\partial z}=0. \tag{2.6}$$

When the carrier starts moving, the initial conditions of the magnetic field are $B_r=0, B_\theta=0$, and $B_z=constant=B_0$ and the boundary conditions of the walls are $v_r|_{r=r_1}=\omega(t)\cdot r_1$, $v_r|_{r=r_2}=\omega(t)\cdot r_2$, where $\omega(t)$ is the time-varying angular velocity, r_1, r_2 are the radii of the inner and outer cylinder, respectively.

2.3. The simplified model

Assuming there is no secondary flow, the axial velocity is much smaller than the circumferential velocity $v_r \ll v_\theta$, and thus $v_\theta^2/r^2=\partial p^*/\partial r$ can be obtained. Due to axial symmetry of the fluid motion, it is represented as $\partial v_\theta/\partial\theta=0$ and $\partial p^*/\partial\theta=0$. If $z\le r$, it derives to $\partial^2 v_\theta/\partial r^2\le\partial^2 v_\theta/\partial z^2$. With the assumption that the magnetic induction is uniform and constant during the whole process, the solution of governing equations can be simplified to

$$\frac{\partial v_\theta}{\partial t}=-\sigma B^2(v_\theta-v)+\eta_f\frac{\partial^2 v_\theta}{\partial z^2},\quad -h/2<z<h/2. \tag{2.7}$$

According to the Laplace transform and Faraday law of electromagnetic induction, finally, the transfer functions of simplified formula can be deduced to

$$\frac{U_{out}(s)}{\omega(s)}=\frac{BrWs}{s+\sigma B^2/\rho+\eta_f/h^2}, \tag{2.8}$$

where U_{out} is the output voltage, $r=\sqrt{r_1^2+r_2^2}$ is the root mean square radius, $W=r_2-r_1$ is the effective width of the fluid channel and h is the height of the fluid channel. The simplified model can meet the non-low-frequency situation (greater than 1 Hz) very well. And there is still room for improving low-frequency performance of this sensor.

3. Analysis by the finite volume method

During the process of establishing a simplified model, the effects of secondary flow, the impact of non-uniform magnetic field and

$$\begin{cases}\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_\theta\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r^2}+v_z\frac{\partial v_r}{\partial z}\right)=\frac{1}{\mu}(B_r\frac{\partial B_r}{\partial r}+B_\theta\frac{\partial B_r}{\partial\theta}-B_\theta^2+B_z\frac{\partial B_r}{\partial z})+\eta_f\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial z^2}\right)-\frac{\partial p^*}{\partial r}\\ \rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\frac{1}{\mu}(B_r\frac{\partial B_\theta}{\partial r}+B_\theta\cdot B_r+B_\theta\frac{\partial B_\theta}{\partial\theta}+B_z\frac{\partial B_\theta}{\partial z})+\eta_f\left(\frac{\partial^2 v_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r^2}+\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right.\\ \left.\quad+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}\right)-\frac{1}{r}\frac{\partial p^*}{\partial\theta}\\ \rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=\frac{1}{\mu}(B_r\frac{\partial B_z}{\partial r}+B_\theta\frac{\partial B_z}{\partial\theta}+B_z\frac{\partial B_z}{\partial z})+\eta_f\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{1}{r^2}\frac{\partial v_z}{\partial\theta}+\frac{\partial^2 v_z}{\partial z^2}\right)-\frac{\partial p^*}{\partial z}\end{cases}, \tag{2.3}$$

and

$$\begin{cases}\frac{\partial B_r}{\partial t}=\frac{B_\theta}{r}\frac{\partial v_r}{\partial\theta}+\frac{v_r}{r}\frac{\partial B_\theta}{\partial\theta}-\frac{B_r}{r}\frac{\partial v_\theta}{\partial\theta}-\frac{v_\theta}{r}\frac{\partial B_r}{\partial\theta}-B_r\frac{\partial v_z}{\partial z}-v_z\frac{\partial B_r}{\partial z}+B_z\frac{\partial v_r}{\partial z}+\frac{1}{\mu\sigma}\left(\frac{\partial^2 B_r}{\partial r^2}+\frac{1}{r}\frac{\partial B_r}{\partial r}+\frac{1}{r^2}\frac{\partial^2 B_r}{\partial\theta^2}-\frac{B_r}{r^2}-\frac{2}{r^2}\frac{\partial B_\theta}{\partial\theta}+\frac{\partial^2 B_z}{\partial z^2}\right)\\ \frac{1}{r}\frac{\partial B_\theta}{\partial t}=B_z\frac{\partial v_\theta}{\partial z}+v_\theta\frac{\partial B_z}{\partial z}-B_\theta\frac{\partial v_z}{\partial z}-v_z\frac{\partial B_\theta}{\partial z}-B_\theta\frac{\partial v_r}{\partial r}-v_r\frac{\partial B_\theta}{\partial r}+B_r\frac{\partial v_\theta}{\partial r}+\frac{1}{\mu\sigma}\left(\frac{\partial^2 B_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial B_\theta}{\partial r}+\frac{1}{r^2}\frac{\partial^2 B_\theta}{\partial\theta^2}-\frac{B_\theta}{r^2}+\frac{2}{r^2}\frac{\partial B_r}{\partial\theta}+\frac{\partial^2 B_\theta}{\partial z^2}\right)\\ \frac{\partial B_z}{\partial t}=B_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial B_r}{\partial r}-B_z\frac{\partial v_r}{\partial r}-v_r\frac{\partial B_z}{\partial r}-\frac{v_\theta}{r}\frac{\partial B_z}{\partial\theta}-\frac{B_z}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_z}{r}\frac{\partial B_\theta}{\partial\theta}+\frac{B_\theta}{r}\frac{\partial v_z}{\partial\theta}+\frac{1}{\mu\sigma}\left(\frac{\partial^2 B_z}{\partial r^2}+\frac{1}{r}\frac{\partial B_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 B_z}{\partial\theta^2}+\frac{\partial^2 B_z}{\partial z^2}\right)\end{cases}. \tag{2.4}$$