Solid-State Electronics 69 (2012) 85–88

Contents lists available at SciVerse ScienceDirect

Solid-State Electronics

journal homepage: www.elsevier.com/locate/sse

Formulas of 1/*f* noise in Schottky barrier diodes under reverse bias

Chan Hyeong Park [a,*], Jong-Ho Lee [b]

[a] Department of Electronics and Communications Engineering, Kwangwoon University, Seoul 139-701, Republic of Korea
[b] School of Electrical Engineering and Computer Science and Inter-university Semiconductor Research Center (ISRC), Seoul National University, Seoul 151-742, Republic of Korea

ARTICLE INFO

Article history:
Received 18 August 2011
Received in revised form 7 November 2011
Accepted 22 November 2011
Available online 23 December 2011

The review of this paper was arranged by Prof. S. Cristoloveanu

Keywords:
1/*f* Noise
Schottky barrier
Diode
Reverse bias
Thermionic emission

ABSTRACT

This paper presents the formulas of 1/*f* noise in Schottky barrier diodes under reverse bias condition. The derived formulas show that the electron density near the metal–semiconductor interface plays an important role in determining the power spectral density of 1/*f* noise current in Schottky diodes under reverse bias. The formulas give information on how to calculate the low-frequency noise in metal–semiconductor (or oxide)–metal devices such as resistive switching devices or memristors which have structures of two back-to-back Schottky barrier diodes as well as in the reverse-biased Schottky diodes.

The formulas show that the power spectral density of the flicker noise in reverse-biased Schottky barrier diodes increases proportional to the square of the dc current in thermionic emission-limited condition.

1. Introduction

Theory of 1/*f* noise in Schottky barrier diodes has been developed over four decades when Schottky barrier diodes are in forward bias region [1–6]. However, the theory and explicit formulas of 1/*f* noise in Schottky barrier diodes under reverse bias have not yet been presented clearly.

Recently, the resistive switching devices and memristors have become one of interesting topics in semiconductor industries as well as academia because these devices can be used as fast nonvolatile devices and new circuit elements [7,8]. Therefore, for the reliable operation of these devices in circuits and systems, an interest on their noise properties has been risen. These devices consist of metal–semiconductor (or oxide)–metal structures and it has become imperative to understand the noise characteristics of Schottky barrier diodes in reverse bias as well as in forward bias regime, because these devices can be considered to be two Schottky barrier diodes connected in series back-to-back.

In this paper, the derivation and formulas of the flicker noise of Schottky diodes in reverse bias are presented in Section 2. The derivation is based upon the thermionic emission–diffusion theory of Schottky barrier diodes [4,9,10]. The derived formulas can be used to calculate and predict 1/*f* noise level in metal–semiconductor–metal or metal–oxide–metal devices. Simulation results and discussion are given in Section 3, and conclusions follow in Section 4.

2. Derivation of the 1/*f* noise formulas

From the thermionic emission theory [11], the dc current of a Schottky barrier diode at a position x_m, where the conduction band edge of a semiconductor side is maximum as shown in Fig. 1, can be written as

$$I = qA[n(x_m) - n_0(x_m)]\,\upsilon_r \tag{1}$$

where q is the magnitude of electronic charge, A is the cross-sectional area of the device, $n(x_m)$ is the steady-state electron density at position $x = x_m$, $n_0(x_m)$ is a quasi-equilibrium electron density at x_m, the density that would occur if it were possible to reach equilibrium without altering the magnitude or position of the potential energy maximum, i.e., $F_n(x_m) = E_{Fm}$ [9,10,12]. The electron surface recombination velocity, υ_r is equal to $\sqrt{k_BT/(2\pi m_n^*)} = A^*T^2/(qN_c)$, where k_B is the Boltzmann constant, T is the absolute temperature, m_n^* is the electron effective mass, A^* is the effective Richardson constant, and N_c is the effective density of states of the conduction band. Taking into account the Schottky barrier lowering, $n_0(x_m)$ can be written as

$$n_0(x_m) = N_c e^{-q\phi_{Bn}/k_BT} = N_c e^{-q(\phi_{Bn0}-\Delta\phi)/k_BT} \tag{2}$$

and $n(x_m)$ is equal to

---

[*] Corresponding author. Tel.: +82 2 940 5133; fax: +82 2 941 8387.
E-mail address: chanpark@kw.ac.kr (C.H. Park).

0038-1101/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.sse.2011.11.030

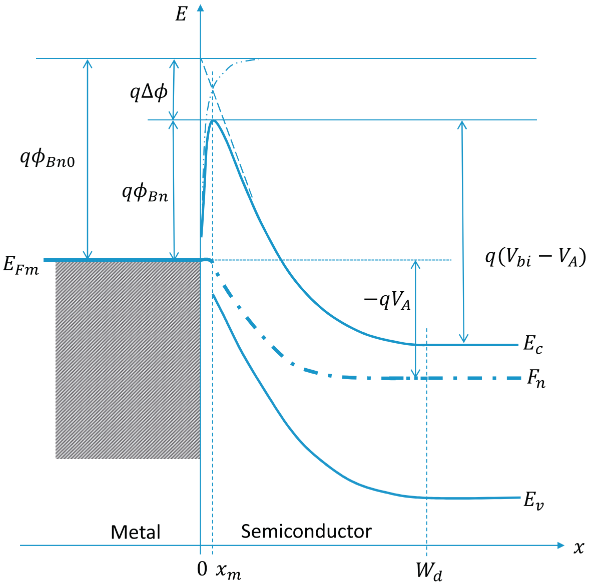

Fig. 1. Energy band diagram of a metal-n-type semiconductor contact under the reverse bias of V_A. F_n denotes the electron quasi-Fermi level in the semiconductor. $q\Delta\phi$ is the Schottky-barrier lowering due to the image charge effect.

$$n(x_m) = N_c e^{-(E_c(x_m)-F_n(x_m))/k_BT} = N_c e^{-q\phi_{Bn}/k_BT} e^{(F_n(x_m)-E_{Fm})/k_BT} \tag{3}$$

where E_c is the conduction band edge, F_n is the electron quasi-Fermi level, and $q\phi_{Bn0}$ is the Schottky barrier energy if the electric field at the surface were zero, and $\Delta\phi(\mathscr{E}_s)$ is the Schottky barrier lowering in volt and is equal to $\sqrt{q|\mathscr{E}_s|/4\pi\epsilon_s}$, where $\mathscr{E}_s$ is the electric field at the metal–semiconductor interface.

With the combined thermionic emission–diffusion theory, the dc current of Eq. (1) can be rewritten as

$$I = qAN_c \frac{v_r v_d}{v_r + v_d} e^{-q\phi_{Bn0}/k_BT} e^{q\Delta\phi/k_BT} \left(e^{qV_A/k_BT} - 1\right) \tag{4}$$

where V_A is an applied bias voltage and v_d is defined to be

$$v_d \equiv \frac{D_n}{\int_{x_m}^{W_d} e^{-(E_c(x_m)-E_c(x))/k_BT} dx} \equiv \frac{D_n}{L_c} \tag{5}$$

where W_d is the depletion-region width, D_n is the electron diffusion constant, and $L_c \equiv \int_{x_m}^{W_d} e^{-(E_c(x_m)-E_c(x))/k_BT} dx$. When the Schottky barrier diode is in reverse bias region with the magnitude of the reverse bias voltage being larger than a few thermal voltages, we can approximate the reverse current as

$$I \simeq -qAN_c \frac{v_r v_d}{v_r + v_d} e^{-q\phi_{Bn0}/k_BT} e^{q\Delta\phi/k_BT}. \tag{6}$$

Here, because $\Delta\phi$ increases as the magnitude $|V_A|$ of the reverse bias voltage increases, the reverse current does not saturate, but actually increases in magnitude as $\ln|I| \propto |V_A|^{1/4}$ for metal–semiconductor contacts, and $\ln|I| \propto |V_A|^{1/2}$ for metal–oxide contacts [8,9]. From the thermionic emission theory, reminding that $n_0(x_m)$ is a constant as a reference value, the low-frequency current fluctuation, $\Delta I(t)$ at x_m can be related to the electron density fluctuation, $\Delta n(x_m,t)$ by

$$\Delta I(t) = qA\Delta n(x_m, t) v_r \tag{7}$$

and this sets the boundary condition for the electron density fluctuation at x_m to be $\Delta n(x_m,t) = \Delta I(t)/qAv_r$. $\Delta n(x_m,t)$ can also be related with the electron quasi-Fermi level fluctuation at x_m, $\Delta F_n(x_m,t)$ as

$$\Delta n(x_m, t) = N_c e^{-\frac{q\phi_{Bn}}{k_BT}} e^{\frac{F_n(x_m)-E_{Fm}}{k_BT}} \frac{\Delta F_n(x_m, t)}{k_BT} = n(x_m) \frac{\Delta F_n(x_m, t)}{k_BT} \tag{8}$$

In the semiconductor region between x_m and W_d which is governed by the drift–diffusion theory, following the procedure of Ref. [4], it can be shown that the current fluctuation in the depletion region of a uniformly-doped n-type semiconductor becomes

$$\begin{aligned}\Delta I(t) = \frac{qD_nA}{L_c} \Bigg[& e^{-\frac{q(V_{bi}-V_A)}{k_BT}} \Delta n(W_d, t) - \Delta n(x_m, t) \\ & + \frac{qN_d}{k_BT} e^{-qV_{bi}/k_BT} \int_{x_m}^{W_d} e^{(F_n(x)-E_{Fm})/k_BT} \Delta\mathscr{E}(x, t) dx \Bigg] \\ & + \frac{1}{L_c} \int_{x_m}^{W_d} e^{-(E_c(x_m)-E_c(x))/k_BT} H_\mu(x, t) dx\end{aligned} \tag{9}$$

where V_{bi} is the built-in voltage of the metal–semiconductor contact, N_d is the donor density, $\Delta n(x,t)$ is the induced electron density fluctuation due to the distributed mobility-fluctuation $1/f$ noise current sources, $\Delta\mathscr{E}$ is the electric field fluctuation, and $H_\mu(x,t)$ is the mobility-fluctuation $1/f$ noise current source and is given by $H_\mu(x,t) = \frac{\Delta\mu_n(x,t)}{\mu_n} I$. Here, it is noted that the electron quasi-Fermi level F_n behaves quite differently between the forward and reverse bias region. In forward bias, F_n is found to be almost flat throughout the depletion region, so that the factor $e^{(F_n(x)-E_{Fm})/k_BT}$ can be placed out of the integral, and use the ac-wise short-circuited condition of $\int_{x_m}^{W_d} \Delta\mathscr{E} dx = 0$ to make the third term of the RHS of Eq. (9) be zero. However, in reverse bias regime, F_n is not found to be flat all across the depletion region [12–14]. Thus, we cannot employ the ac-short circuit condition in a rigorous way. An assumption is needed to neglect the third term in Eq. (9), so that the current fluctuation inside the depletion region is approximated by

$$\Delta I(t) \approx -\frac{qD_nA}{L_c} \Delta n(x_m, t) + \frac{1}{L_c} \int_{x_m}^{W_d} e^{-(E_c(x_m)-E_c(x))/k_BT} H_\mu(x, t) dx. \tag{10}$$

$\Delta n(x_m,t)$ in the first term on the RHS of Eq. (10) represents the induced electron density fluctuation due to the distributed $H_\mu(x,t)$ noise sources.

Inserting the electron density fluctuation from thermionic emission at $x = x_m$ (cf. Eq. (7)) into the low-frequency current fluctuation from the drift–diffusion region ($x_m < x \leqslant W$) of Eq. (10), we find that

$$\Delta I(t) = \frac{v_r}{v_r + v_d} \frac{1}{L_c} \int_{x_m}^{W_d} e^{-(E_c(x_m)-E_c(x))/k_BT} H_\mu(x, t) dx. \tag{11}$$

Finally, by using $S_{H_\mu}(x, x', f) = \frac{\alpha_H}{fn(x)} \delta(x - x')$, the power spectral density, $S_I(f)$ of the current fluctuation, is then found to be

$$S_I(f) = \left(\frac{v_r}{v_d + v_r}\right)^2 \frac{1}{L_c^2} \int_{x_m}^{W_d} \frac{\alpha_H I^2}{fn(x)A} e^{-2(E_c(x_m)-E_c(x))/k_BT} dx \tag{12}$$

where f is the frequency and α_H is the Hooge parameter.

3. Simulatioin results and discussion

To calculate the power spectral density of current noise in Schottky barrier diodes under reverse bias, let us assume the depletion-region approximation for a uniformly-doped semiconductor. Then, $E_c(x_m) - E_c(x)$ is of parabolic form:

$$E_c(x_m) - E_c(x) = \frac{q^2N_d}{\epsilon_s} \left(W_d x - \frac{x^2}{2}\right) \tag{13}$$

where ϵ_s is the electrical permittivity of the semiconductor. Since the only explicitly unknown quantity in Eq. (12) is the electron density $n(x)$, it is needed to have $n(x)$ in an analytic form. It can be shown that $n(x)$ has a following form:

$$\begin{aligned}n(x) = N_d e^{-\frac{1}{2}\left[\sqrt{\frac{2q(V_{bi}-V_A)}{k_BT}} - \left(\frac{x}{L_D}\right)\right]^2} + N_d e^{-\frac{qV_{bi}}{k_BT}} \left(\frac{v_r}{v_r + v_d}\right) \\ \times 2\sqrt{\frac{q(V_{bi} - V_A)}{k_BT}} D(z) \left(1 - e^{qV_A/k_BT}\right)\end{aligned} \tag{14}$$