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Discussions and extension of van Vliet's noise model for high speed bipolar transistors

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The general van Vliet noise model for transistors was derived for base minority carriers only under adiabatic boundary conditions. This paper extends the model by including the emitter minority carrier noise and the base–collector space charge region (CB SCR) effects based on the mathematical framework developed by van Vliet. Both the finite surface recombination velocity at polysilicon emitter and the finite carrier exit velocity at CB SCR are considered. It is proved that the van Vliet model can be directly used to include the emitter minority carrier noise, and the model holds when the two finite velocity boundary conditions are imposed. A new set of equations are derived to include the effect of CB SCR delay on noise.

1. Introduction

General noise transport theory for PN junction-like devices was developed by van Vliet in [1] based on 3-D microscopic noise transport equation or Langevin equation. Specifically, the neutral *base* minority carrier noise of a PNP transistor was solved. The extremely useful result is that the base minority carrier noise induced base and collector current noises can be related to the *Y*-parameters of *base* region at low injection levels. The original result is in common-base configuration, and can be transformed into common-emitter configuration as [2]

$$S_{ib}^B = 4kT\Re(Y_{11}^B) - 2qI_b^B, \quad S_{ic}^B = 4kT\Re(Y_{22}^B) + 2qI_c \approx 2qI_c,$$
$$S_{icib^*}^B = 2kT(Y_{21}^B + Y_{12}^{B*} - g_m) \approx 2kT(Y_{21}^B - g_m), \quad (1)$$

where g_m is the transconductance at low frequency, and I_b^B is the DC neutral base recombination current. The superscript *B* refers to base. (1) is referred as van Vliet noise model and is widely used [3–6]. In all existing applications of (1), the *Y*-parameters of the whole intrinsic transistor are used, and the total base current I_B is used in place of the base recombination current I_b^B, which is negligible in modern transistors, without justification. This work is aimed to extend the model to the whole transistor by including emitter minority carrier noise and CB space charge region (CB SCR) effect for modern high speed transistors. Note that there is no simple general relation between noise and terminal parameters at high injection levels, which is beyond the scope of this work.

Now we review briefly the equations and boundary conditions used in [1]. Fig. 1 sketches the structure of the PNP transistor. S_α and S_β are the neutral base ending surfaces at the EB and CB junctions. S_f is the free surface of the base. S_c is the base contact surface. The frequency domain Langevin equation for the base minority carriers, the holes, is

$$L_p p = \xi(r,s), \quad \xi(r,s) = \zeta(r,s) + \nabla\cdot\gamma(r,s), \quad (2)$$

$$L_p = s + 1/\tau_p + \nabla\cdot\mu\vec{E} - D\nabla^2, \quad s = j\omega. \quad (3)$$

Here p is the hole density fluctuation caused by noise source $\xi(r,s)$. $\vec{E}$ is the base built-in electric field and can be position dependent. τ_p is the base hole lifetime and can be position dependent. μ and D are the base hole mobility and diffusion coefficient, both of which are assumed to be position independent. $\gamma(r,s)$ is the diffusion noise source (a flux vector) with its power spectrum density (PSD) given by

$$S_\gamma(r,r') = 4Dp_s(r)\delta(r-r')\mathbb{I}, \quad (4)$$

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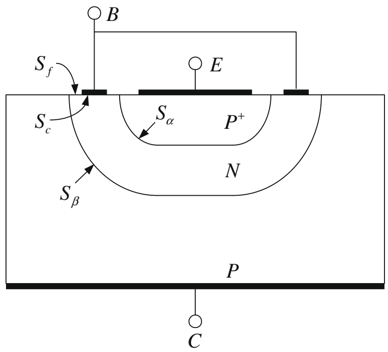

Fig. 1. Schematic geometry of a PNP transistor.

where $p_s(r)$ is the DC hole density at position r for given bias and $\mathbb{I}$ is the unit tensor of rank two. $\zeta(r,s)$ is the GR noise source with its PSD given by

$$S_\zeta(r,r') = \left\{\frac{4p_s(r)}{\tau_p} - 2D\nabla^2 p_s(r) + 2\nabla\cdot[\mu\vec{E}(r)p_s(r)]\right\}\delta(r-r'). \quad (5)$$

In [1], three types of boundary conditions were used: S_α, S_β both shorted (admittance configuration); S_α, S_β both opened (impedance configuration); S_α opened and S_β shorted (hybrid configuration). The results are fully equivalent and can be converted from one to another through two-port transformation. Here we only discuss the first case. [1] assumed zero carrier fluctuation once a junction is shorted, the so-called adiabatic boundary condition, explicitly,

$$p|_{\sigma=S_\alpha} = 0, \quad p|_{\sigma=S_\beta} = 0. \quad (6)$$

Below we show the problems found for the van Vliet model when applied to modern transistors:

- For early transistors, the total base current is dominated by the base recombination current I_b^B. However, for high speed transistors such as SiGe HBTs, the base current is dominated by the injection of base minority carriers back into the emitter, I_b^E, instead of I_b^B due to device scaling and process enhancement [7]. I_b^B is typically small as base width is much less than diffusion length. Therefore, the van Vliet model should be extended to include the emitter minority carrier noise, S_{ib}^E. This is readily achievable from [1] due to its generality. However, it is unknown how the frequency dependences of S_{ib}^E and S_{ib}^B differ and whether it is necessary to model the input non-quasi-static (NQS) effect for base [8].
- Polysilicon emitter contact is commonly used in modern transistors to obtain reliable shallow EB junction [9]. One of the significant features is the high current gain attributed to the finite effective surface recombination velocity v_{sr} at the poly-crystal interface for emitter minority carriers. However, finite v_{sr} is not considered when [1] is used to calculate S_{ib}^E.
- Due to the strong electrical field in CB junction, the base minority carriers exit the base with a finite velocity v_{ex} [10]. Therefore the Dirichlet boundary condition $p|_{\sigma=S_\beta} = 0$ in (6) is inappropriate and should be replaced with a Neumann boundary condition, to which the van Vliet model is not justified. Note this problem is essentially the same as the former finite v_{sr} problem.
- For scaled bipolar transistors, the CB space charge region carrier transport becomes more significant than the base electron transport in determining f_T. This has noticeable impact on noise as well [11]. However it is not included in the van Vliet model.

In this work, we extend the van Vliet model by solving the above problems. Drift-diffusion model for carrier transport is used. We still use the PNP transistor to derive our results to better follow the mathematical framework developed by van Vliet. The results are general and applicable to NPN transistors. In Section 2, we will show that the van Vliet model can automatically include the emitter minority carrier noise and is valid with finite surface recombination velocity in poly emitter transistors. In Section 3, we will prove that the van Vliet model holds when the finite exit velocity at CB junction is imposed. In Section 4, the impacts of v_{sr} and v_{ex} on S_{ib}^E and S_{ib}^B are illustrated using an 1-D ideal SiGe HBT. In Section 5, a new formula is derived to include the collector transit time effect.

2. Emitter minority carrier noise

To obtain the PSD of the emitter minority carrier noise contributing to the base current, S_{ib}^E, we need to solve a 3-D Langevin equation for the emitter minority carriers, electrons for PNP. The equation can be obtained by replacing p with n and $\vec{E}$ with $-\vec{E}$ for Eqs. (2)–(5). We first consider the surface recombination velocity v_{sr} as infinite, the result can be directly obtained from van Vliet's derivation. Then we extend the result for finite v_{sr}.

2.1. Infinite surface recombination velocity

With $v_{sr} \to \infty$, the boundary condition for electron fluctuation at the emitter contact is $n = 0$. $n = 0$ at the EB junction surface of the emitter as we assume adiabatic boundary condition. With such a set of boundary conditions, the Langevin equation for the emitter minority electrons resembles the Langevin equation for the base minority holes in (2)–(6). The emitter minority carrier induced base noise current i_b^E, is in analogy similar to the base minority carrier induced emitter noise current i_e^B. Therefore the PSD of i_b^E takes the functional form of the PSD of i_e^B in [1], that is,

$$S_{ib}^E = 4kT\Re(Y_{11}^E) - 2qI_b^E, \quad (7)$$

where Y_{11}^E is the input admittance seen by the base terminal due to emitter electron injection. I_b^E is the electron current at the emitter injection point, essentially the mount of base current due to the injection of base electrons. At the low frequency limit $S_{ib}^E \approx 2qI_b^E$ since $Y_{11}^E \approx qI_b^E/kT$.

The emitter electron density fluctuations induce emitter hole density fluctuations to maintain quasi-neutrality due to dielectric relaxation. The electron density fluctuations, however, induce hole current fluctuation only at the emitter contact but not at the EB junction, because holes are the majority carriers in emitter. Therefore the emitter electron noise only contributes to base current noise i_b but not collector current noise i_c. This is an important difference between the emitter minority carrier noise and the base minority carrier noise. The PSDs of total i_b and i_c can then be obtained as

$$S_{ib}^{EB} = 4kT\Re(Y_{11}^{EB}) - 2qI_b, \quad S_{ic}^{EB} = 4kT\Re(Y_{22}^{EB}) + 2qI_c \approx 2qI_c,$$
$$S_{icib^*}^{EB} = 2kT(Y_{21}^{EB} + Y_{12}^{EB*} - g_m) \approx 2kT(Y_{21}^{EB} - g_m), \quad (8)$$

where

$$Y_{11}^{EB} = Y_{11}^E + Y_{11}^B, \quad I_b = I_b^E + I_b^B,$$
$$Y_{21}^{EB} = Y_{21}^B, \quad Y_{22}^{EB} = Y_{22}^B, \quad Y_{12}^{EB} = Y_{12}^B.$$

(8) has the same functional form as (1), meaning that the van Vliet model equations in (1) can be directly applied to include emitter hole noise by simply replacing Y^B with Y^{EB}, the Y-parameters due to minority carrier transport in both base and emitter regions. Note (8) holds when Y^{EB} includes the EB depletion capacitance C_{te}, since C_{te} does not contribute to $\Re(Y_{11}^{EB})$ hence S_{ib}^{EB}, which is consistent with the fact that C_{te} is noiseless.