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Attractive method for thickness-independent permittivity measurements of solid dielectric materials

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ABSTRACT

We propose a non-resonant-type method for reference-plane invariant and thickness-independent permittivity measurement of solid dielectric materials when there is no information from permittivity. To achieve this, we derive a branch-index-independent function and verify that it determines the sample thickness and permittivity without resorting to finding the correct branch index value for three different measurement scenarios: (a) measurements at closely separated frequencies of non-dispersive samples, (b) measurements at largely separated frequencies of non-dispersive samples, and (c) measurements at largely separated frequencies of dispersive samples. Via a differential uncertainty analysis, the effect of inaccurate sample thickness on permittivity determination is also presented for demonstration of the improvement achieved by our method. For validation, we measured the permittivity of an approximately 20 mm PTFE sample using the presented method and compared its result with those from other similar methods. From the comparison, we note that the accuracy of tested methods from literature drastically decreases when accurate information of sample thickness is not known (in addition to a further decrease in accuracy when correct transformation factors from reference-planes are not known a priori), while the accuracy of our method is not influenced.

1. Introduction

Materials can in general be described by their electromagnetic properties [1,2]. These properties are associated with the response of a material to the stimulus of electric and magnetic origins. While magnetic materials are characterized electrically by their relative complex permittivity (ε_r) and relative complex permeability (μ_r), dielectric materials are electrically characterized by only ε_r. In addition to conventional dielectric or magnetic materials, the new trend is to fabricate new type of materials by periodically arranging metal-dielectric composites [3–5]. These new type of materials, named as metamaterials (MMs) or left-handed materials, have appealing electromagnetic features which can be utilized in many applications such as perfect lens [4] and invisibility cloak [6] due to their negative refraction properties [7] unachievable in natural materials.

For characterization of conventional materials and engineering-based MMs, different methods have been proposed in the literature. Among these methods such as open-ended probes [8,9], resonant-type methods [10–14], non-resonant-type (transmission-reflection, transmission-only, or reflection-only) methods [14–66], non-resonant-type methods are generally utilized for broadband measurements.

Non-resonant-type methods generally require that the reference-planes (RPs) and measurement planes (MPs) coincide with each other. The techniques using amplitude-only measurements [23–25], [39–42] as well as RP invariant (RPI) expressions [46–50] can be utilized to eliminate this requirement. While the amplitude-only techniques are accurate, their expressions are relatively involved for ε_r determination. The methods [46–50] can be applied to remove the above drawback of the methods in [23–25], [39–42] by using relatively simple RPI expressions. While the techniques in [46–48] are suitable for ε_r retrieval of solid materials, those in [49] and [50] can be utilized for ε_r determination of granular and/or liquid materials.

Another important problem of non-resonant-type methods is the requirement of an initial guess or seed for ε_r and/or μ_r in

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their accurate and unique retrieval. This is especially important for new fabricated materials whose electromagnetic properties are not known properly [31]. There are some techniques, such as the phase-unwrapping method [32,33,57], the stepwise method [30,43], the group-delay technique [19], the method based on Kramers–Kronig relations [52,60,61], and methods based on measurements at multiple-frequencies [34–38], to determine an initial guess for ε_r and/or μ_r of materials. Among these methods, while the phase-unwrapping and stepwise methods extract ε_r and/or μ_r when correct branch index value at a specific reference point is known, the group-delay technique retrieves ε_r and/or μ_r of materials whose electromagnetic properties vary slowly with frequency (non-dispersive materials). Besides, the methods based on Kramers–Kronig relations utilize the causality property in the extraction of ε_r and/or μ_r of dispersive and non-dispersive materials provided that measurements have been carried out over a broad frequency band. In addition, methods based on measurements at multiple-frequencies extract ε_r and/or μ_r of materials provided that dispersion model for the material is somewhat known a priori or provided that material is of non-dispersive type. Considering all these methods, we note that none of them can be utilized for ε_r and/or μ_r measurements of dispersive low-loss or medium-loss materials over a narrow frequency band when no information of correct branch index value is known at any frequency.

Besides, non-resonant-type methods commonly necessitate the information of the sample thickness. Differently from the case that RPI techniques can be utilized for relaxing the need that RPs and MPs overlap one another, many of the non-resonant-type techniques in [15–66] require the thickness information of the sample. Spectroscopic ellipsometry [67], spectral reflectometry/interferometry [68–71], wavelength scanning [72–74], and full spectra fitting techniques [75–77] can be utilized for determining the sample thickness. Ellipsometry technique necessitates measurements at different angles and in turn may not be so accurate if angle arrangement is not so precise. Besides, spectral reflectometry/interferometry method is based on extremes and/or envelopes of reflectance and/or transmittance and can be applied only for non-dispersive materials. On the other hand, wavelength scanning technique needs a wide frequency range to determine sample thickness in addition to ε_r and/or μ_r. Finally, full spectra technique requires an information about the behavior ε_r and μ_r to model their wavelength/frequency dependence. While spectroscopic ellipsometry, spectral reflectometry/interferometry, and wavelength scanning methods necessitate a broad frequency band for thickness measurements, full spectra fitting technique requires some knowledge on behavior of material dispersive character. We discuss in detail in Sections 5 and 6 that sample thickness, which plays a key role in multiple-reflection phenomenon occurring in many low-loss or lossless materials, is a crucial parameter in retrieval of electromagnetic properties of materials.

In this research paper, as discussed in previous paragraphs, we propose a non-resonant-type method to eliminate many of the disadvantages of non-resonant-type techniques in the literature for ε_r measurements of solid dielectric materials using RPI expressions when their thickness is not known and when there is no information available for the range and for the dispersive character of ε_r.

2. Permittivity determination

The problem of ε_r determination of a solid dielectric sample with length L inside a measurement cell (waveguide) is depicted in Fig. 1. It is assumed that an appropriate calibration is performed to RPs and that the sample is linear, isotropic, and homogeneous and fills entirely the cross section of the cell. Forward and backward reflection and transmission scattering (S-) parameters at RPs [20]

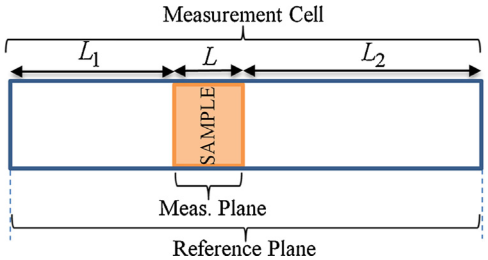

Fig. 1. The measurement configuration for reference-plane invariant and thickness-independent ε_r determination of solid materials.

$$S_{11} = R_1^2 \frac{\Gamma(1-T^2)}{1-\Gamma^2T^2}, \quad S_{22} = R_2^2 \frac{\Gamma(1-T^2)}{1-\Gamma^2T^2}, \tag{1}$$

$$S_{21} = S_{12} = R_1R_2 \frac{T(1-\Gamma^2)}{1-\Gamma^2T^2}. \tag{2}$$

Here, Γ and T are, respectively, the first reflection coefficient and the propagation factor, and R_1 and R_2 are the RP factors. Their expressions for the harmonic time dependence $\exp(+i\omega t)$ read

$$T = \exp(-ik_0\chi L), \quad \Gamma = \frac{\kappa-\chi}{\kappa+\chi}, \quad \chi = \sqrt{\varepsilon_r - (f_c/f)^2}, \tag{3}$$

$$R_u = \exp(-ik_0\kappa L_u), \quad \kappa = \sqrt{1-(f_c/f)^2}, \quad u = 1, 2, \tag{4}$$

where f, f_c, and k_0 are the operating and cutoff frequencies and the free-space wavenumber, respectively, and ε_r is the relative permittivity of the sample. It is seen from Eqs. (1)–(4) that ε_r is a function of R_1, R_2, and L through the measured S_{11}, S_{21}, S_{22} where the distances between RP planes and sample end faces are unknown (R_1 and R_2 are not known).

At this point, our aim is to derive expressions for RPI thickness-independent ε_r measurement of solid samples. Toward this end, we first define new variables

$$A = \frac{S_{11}S_{22}}{S_{21}S_{12}} = \frac{\Gamma^2(1-T^2)^2}{T^2(1-\Gamma^2)^2}, \tag{5}$$

$$B = S_{21}S_{12} - S_{11}S_{22} = \exp[-2ik_0\kappa(L_{air} - L)] \left(\frac{T^2-\Gamma^2}{1-\Gamma^2T^2} \right), \tag{6}$$

where $L_{air} = L_1 + L + L_2$ is the distance between RPs in Fig. 1 and assumed to be known a priori. It is seen from Eqs. (5) and (6) that both expressions are RPI (i.e., they are independent of L_1 and L_2). While Eq. (5) is identical to that in [20,46,48–50], Eq. (6) is slightly different and suitable for us for thickness-independent measurements.

Second, we extract T^2 in terms of Γ from Eq. (5)

$$T^4 - \xi T^2 + 1 = 0, \quad \xi = 2 + A(\Gamma - 1/\Gamma)^2, \tag{7}$$

$$T^2_{(1,2)} = \left[\xi \mp \sqrt{\xi^2 - 4}\right]/2. \tag{8}$$

It is seen from Eq. (8), there are two roots of T^2. In next section, both the necessary as well as sufficient criteria for selecting the correct sign from Eq. (8) will be discussed in more detail.

Third, assuming that the correct value of T^2 is determined from Eq. (8), we find L from Eq. (3)

$$L = [i\ln(T^2) \pm 2\pi m]/(2k_0\chi), \quad m = 0, 1, 2, \ldots, \tag{9}$$