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An analytic model for threshold voltage shift due to quantum confinement in surrounding gate MOSFETs with anisotropic effective mass

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ABSTRACT

Threshold voltage shift due to quantum confinement in surrounding gate MOSFETs with anisotropic effective mass is calculated from the solution of 2D Schrödinger equation in elliptic coordinates. The solutions are of the Mathieu function type. It is shown that for some intermediate range of radius in silicon, several subbands need be taken into account to obtain accurate threshold voltage shifts. However, for small radius, only the lowest subband need be considered, and even anisotropic effective mass can be replaced by reduced isotropic effective mass, for which Bessel function is sufficient.

1. Introduction

To extend CMOS scaling down to 10 nm gate length and beyond [1], multiple-gate (MG) MOSFETs have been intensively and extensively studied because of their higher current drive and better short-channel characteristics [2]. In addition to short-channel effects (SCEs) [3], quantum-mechanical (QM) effects play more and more important roles in aggressively scaled devices, especially those with nanowire-like Si body (i.e., 2D quantum confinement), such as triple-gate (TG), quadruple-gate (QG), and surrounding-gate (SG) MOSFETs [2]. QM effects manifest themselves in two ways [3]. First, the slope of inversion charge versus gate voltage curve is degraded because the inversion charge distribution is pushed further into Si, i.e., away from the oxide interface. Second, the threshold voltage (V_t) is shifted to a higher value due to the higher quantized subband energies. Previously, the QM V_t shift has been analytically modeled for conventional bulk MOSFETs [3], double-gate (DG) MOSFETs [4], and also TG and QG MOSFETs [5]. In this paper, we develop an analytic model for quantum confinement induced V_t shift in undoped SG MOSFETs, a type of MG MOSFETs with circular cross-sections. To deal with the ellipsoidal constant energy surfaces of the Si conduction band, which are induced by anisotropic effective mass, we use elliptic coordinates, where Mathieu functions are invoked [6].

2. Analytic model

Following the general method of Stern and Howard [7] for films, Bescond et al. [8] have proposed the effective-mass approach for n-type nanowire MOSFETs with arbitrary orientation very recently. To model the V_t shift caused by quantum confinement in Si nanowire MOSFETs, it is reasonable to make use of Boltzmann statistics as well as parabolic approximation in the weak inversion region. Furthermore, the mobile charge term in the Poisson equation can be neglected, leading to the decoupling of Poisson's and Schrödinger equations. Therefore, in the undoped or lightly-doped body, the potential is almost constant [9], and we can assume the 2D quantum well to be flat-bottomed with infinitely high potential barriers.

For an arbitrarily oriented undoped or lightly-doped cylindrical SG MOSFET, whose schematic cross-section diagram is illustrated in Fig. 1, the total energy under weak inversion condition is given by

$$E = E' + \frac{\hbar^2 k_y^2}{2m_d} \tag{1}$$

where E' is the bottom energy of each discrete subband, determined by the eigenvalue equation

$$\left(-\frac{\hbar^2}{2m_x}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m_z}\frac{\partial^2}{\partial z^2} + V(x,z)\right)\zeta(x,z) = E'\zeta(x,z) \tag{2}$$

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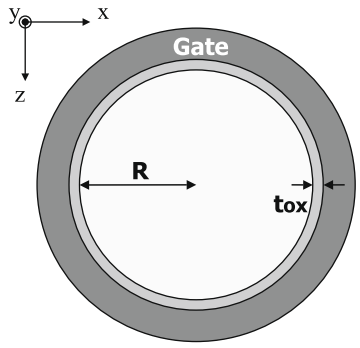

Fig. 1. Schematic cross-section diagram of an SG MOSFET.

with

$$V(x,z)=\begin{cases}0, & \sqrt{x^2+z^2}<R\\ \infty, & \sqrt{x^2+z^2}\geqslant R\end{cases} \tag{3}$$

Here, m_d is the density-of-state (DOS) effective mass. According to (2), we have chosen a (x,z) coordinate to diagonalize the reduced 2×2 reciprocal effective-mass tensor. m_x and m_z are the corresponding effective masses in the x- and z-directions, respectively. We choose the coordinates such that $m_x \geqslant m_z$.

For the special case of $m_x = m_z$, eigenvalues of E' can be easily obtained with Bessel functions [10]:

$$E' \triangleq E_{mn} = \frac{2\hbar^2 q_{mn}}{m_x R^2} \tag{4}$$

where q_{mn} is determined by the boundary condition

$$J_m(2\sqrt{q_{mn}}) = 0 \quad \text{for } m = 0,1,2,\ldots \tag{5}$$

Here, $J_m(\rho)$ denotes the Bessel function of the first kind. The subscript n indicates that $2\sqrt{q_{mn}}$ is the nth zero of $J_m(\rho)$. There is an extra two-fold degeneracy for $m \neq 0$.[1]

For the general case of $m_x > m_z$, by making a coordinate scaling: $X = \sqrt{m_x/m_0}x$ and $Z = \sqrt{m_z/m_0}z$, where m_0 is the free electron mass, (2) can be transformed to

$$\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Z^2}+\frac{2m_0E'}{\hbar^2}\right)\zeta(X,Z)=0 \tag{6}$$

with the boundary condition

$$\zeta(X,Z)=0, \quad \text{where } \sqrt{\frac{m_0}{m_x}X^2+\frac{m_0}{m_z}Z^2}=R. \tag{7}$$

The new boundary is an ellipse instead of a circle. Therefore, to solve the eigenvalue equation (2) generally, we need to use the elliptic coordinates, and deal with Mathieu functions based on separation of variables. The solution in the elliptic coordinates is given by [6]

$$\zeta(\xi,\eta)=\sum_{m=0}^{\infty}A_m C_m(\xi,q)c_m(\eta,q)+\sum_{m=1}^{\infty}B_m S_m(\xi,q)s_m(\eta,q) \tag{8}$$

where the elliptic coordinates (ξ,η) is corresponding to (X,Z) by

$$X = a\cosh\xi\cos\eta \tag{9}$$

$$Z = a\sinh\xi\sin\eta \tag{10}$$

with a defined as the focal distance of the elliptical boundary given by

$$a = R\sqrt{\frac{m_x-m_z}{m_0}}. \tag{11}$$

Note that we choose $m_x > m_z$. In Eq. (8), A_m and B_m are coefficients; $c_m(\xi,q)$ and $s_m(\xi,q)$ are the Mathieu functions with order m of cosine and sine types, respectively; $C_m(\xi,q)$ and $S_m(\xi,q)$ are the modified Mathieu functions with order m of cosine and sine types, respectively.[2] Boundary condition yields

$$C_m\left(\xi_b, q_{mn}^{(c)}\right)=0 \quad \text{for } m=0,1,2,\ldots \tag{12}$$

or

$$S_m\left(\xi_b, q_{mn}^{(s)}\right)=0 \quad \text{for } m=1,2,3,\ldots \tag{13}$$

Here, ξ_b is defined by $\xi_b = \cosh^{-1}\sqrt{m_x/(m_x-m_z)}$. The subscript n indicates that ξ_b is the nth zero of $C_m(\xi,q_{mn}^{(c)})$ and $S_m(\xi,q_{mn}^{(s)})$. Energy is related with $q_{mn}^{(l)}$ ($l = c$ or s) through the following equation

$$E' \triangleq E_{mn}^{(l)} = \frac{2\hbar^2 q_{mn}^{(l)}}{(m_x-m_z)R^2} \tag{14}$$

Within the framework of quantum mechanics, the total mobile charge density per unit gate length N^{QM} is given by

$$N^{QM} = N^{1D}\exp\left(\frac{E_f-E_s^{QM}}{k_BT}\right) \tag{15}$$

with

$$N^{1D}=\sum_k N_k^{1D} \tag{16}$$

and

$$N_k^{1D}=\begin{cases}\sum_m\sum_n g\sqrt{\frac{2m_dk_BT}{\pi\hbar^2}}\exp\left(-\frac{E_{mn}^{(c)}}{k_BT}\right)\\ \quad+\sum_m\sum_n g\sqrt{\frac{2m_dk_BT}{\pi\hbar^2}}\exp\left(-\frac{E_{mn}^{(s)}}{k_BT}\right), & m_x>m_z\\ \sum_m\sum_n gD\sqrt{\frac{2m_dk_BT}{\pi\hbar^2}}\exp\left(-\frac{E_{mn}}{k_BT}\right), & m_x=m_z\end{cases} \tag{17}$$

Here, E_s^{QM} is the quantum-mechanically calculated conduction band edge. The subscript k denotes valley index. g and m_d are the degeneracy and the DOS effective mass, respectively. $E_{mn}^{(l)}$ and E_{mn} are the subband energies for the cases of $m_x > m_z$ and $m_x = m_z$, which can be solved through Mathieu and Bessel functions, respectively. The coefficient D is responsible for the degeneracy which comes from symmetry: $D = 1$ for $m = 0$; $D = 2$ otherwise. Without QM effects, the classical theory leads to

$$N^{CL} = \pi R^2 N^{3D}\exp\left(\frac{E_f-E_s^{CL}}{k_BT}\right) \tag{18}$$

where E_s^{CL} is the classical counterpart of E_s^{QM}, and N^{3D} is the 3D effective DOS, i.e., $N^{3D} = N_c$ for the conduction band. Equating N^{QM} in (16) to N^{CL} in (18) yields an analytic model for the shift of the QM potential compared to the classical one:

$$\Delta\psi_s^{QM} = \frac{E_s^{QM}-E_s^{CL}}{-q} = \frac{k_BT}{q}\ln\left(\frac{\pi R^2N^{3D}}{N^{1D}}\right). \tag{19}$$

Knowing $\Delta\psi_s^{QM}$, one can easily calculate the V_t shift due to the QM effects as [3]

$$\Delta V_t^{QM} = \frac{S}{60}\Delta\psi_s^{QM} \tag{20}$$

[1] The twofold degeneracy is a result of symmetry. Actually, we have another group of characteristic values determined by $J_{-m}(2\sqrt{q'_{mn}})=0$, where $m=1,2,\ldots$ Based on the circular symmetry, it is not surprising that $q'_{mn}=q_{mn}$, which finally leads to the twofold degeneracy.

[2] The Mathieu and the modified Mathieu functions with order m of cosine (sine) type may be denoted in some references as $ce_m(\xi,q)$ ($se_m(\xi,q)$) and $Ce_m(\xi,q)$ ($Se_m(\xi,q)$), respectively.