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Generalised spatial and spatiotemporal autoregressive conditional heteroscedasticity

Philipp Otto *, Wolfgang Schmid, Robert Garthoff

European University Viadrina, Große Scharrnstraße 59, 15230 Frankfurt (Oder), Germany

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ABSTRACT

In this paper, we introduce a new spatial model that incorporates heteroscedastic variance depending on neighbouring locations. The proposed process is considered as the spatial equivalent to the temporal autoregressive conditional heteroscedasticity (ARCH) model. We also show how the newly introduced spatial ARCH model can be used in spatiotemporal settings. In contrast to the temporal ARCH model, in which the distribution is known given the full information set for the prior periods, the distribution is not straightforward in the spatial and spatiotemporal setting. However, the model parameters can be estimated using the maximum-likelihood approach. Via Monte Carlo simulations, we demonstrate the performance of the estimator for a specific spatial weighting matrix. Moreover, we combine the known spatial autoregressive model with the spatial ARCH model assuming heteroscedastic errors. Eventually, the proposed autoregressive process is illustrated by an empirical example. Specifically, we model lung cancer mortality in 3108 U.S. counties and compare the newly introduced model with four benchmark approaches.

1. Introduction

Various specifications for spatial autoregressive (SAR) models have been proposed in the past and current literature (cf. Anselin, 2010). In particular, the spatial models introduced by Whittle (1954) were extended to incorporate external regressors (see, e.g., Elhorst, 2010 for an overview) and autocorrelated residuals (e.g., Fingleton, 2008a), respectively. Currently, these spatial models are

* Corresponding author.
E-mail address: potto@europa-uni.de (P. Otto).

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widely implemented in statistical software packages, making it simple to model spatial clusters of high and low observations. Consequently, a wide range of applications can be found in empirical research, including econometrics (e.g., Holly et al., 2010; Fingleton, 2008b), biometrics (e.g., Mateu et al., 2010; Shinkareva et al., 2006; Ho et al., 2005; MacNab and Dean, 2001) and environmetrics (e.g., Fassò and Finazzi, 2011; Fassò et al., 2007; Mateu et al., 2007; Fuentes, 2001).

However, spatial models that assume spatially dependent second-order moments, such as the well-known autoregressive conditional heteroscedasticity (ARCH) and generalised ARCH (GARCH) models in time-series analysis proposed by Engle (1982) and Bollerslev (1986), have not been previously discussed. Borovkova and Lopuhaa (2012) and Caporin and Paruolo (2006) introduced a temporal GARCH model, which includes temporal lags influenced by neighbouring observations. For the two-dimensional setting, Bera and Simlai (2005) proposed a special type of a spatial ARCH (spARCH) model, the SARCH(1) process, that results from employing the information matrix (IM) test statistic in a simple SAR model. Furthermore, Noiboar and Cohen (2005, 2007) introduced a multidimensional GARCH process to detect image anomalies. However, these recent extensions considered only special approaches, and no general model has been presented. Moreover, there has been no strict analytical examination of the introduced models, and the generalisation of an ARCH or GARCH model to the multidimensional setting does not seem to be straightforward.

For financial time series such as stock returns, ARCH and GARCH processes are widely used because they can describe the time-dependent risk behaviour of an asset. Most financial assets can be traded worldwide almost without time delay at stock exchanges (see, e.g., Shkilko and Sokolov, 2016), so that high spatial dependence is not expected in the risk behaviour of an asset. However, such spatial dependence might occur in markets that are constrained in space. A typical example of such markets is the real-estate market. The prices for real estate depend strongly on the location of the property. Much like stock markets, real-estate markets are also characterised by uncertainty due to the influences of many factors, most of which are not measurable (see also Tuzel and Zhang, 2017). Considering the temporal changes of real-estate prices (spatiotemporal data) or the differences between real-estate prices and standard ground values (spatial data), one would expect conditional spatial heteroscedasticity. The mean of these changes/differences lies near zero, and there are spatial clusters of large and small variance. In urban regions, we are typically faced with heterogeneous types of buildings and a high demand for real estate combined with a limited supply. Thus, the variance is higher than in rural regions, where land and buildings are rather homogeneous. Obviously, the variance at one location depends on the variances at the neighbouring locations, if it is possible to commute between these places. A further example is the level of salary, which generally depends on the level of qualification. We expect high volatility in salaries in highly populated regions, since the diversity or heterogeneity of jobs increases with increasing urban density. In addition, we want to note that conditional heteroscedasticity may result from unobservable factors, as is also the case for the returns of financial assets. The proposed spARCH process is a powerful tool for modelling such spatially dependent heterogeneity, as one might assume the error term of any regression model to follow this kind of ARCH process. Thus, we show in the ensuing sections how it is possible to estimate the ARCH-type errors of SAR models.

In contrast to nonstationary spatial models (see, e.g., Sampson and Guttorp, 1992; Fuentes, 2001, 2002; Schmidt and O'Hagan, 2003; Heaton et al., 2017; Ombao et al., 2008; Stroud et al., 2001), our paper proposes a spatial process with similar properties to the temporal ARCH process introduced by Engle (1982), i.e., conditional autoregressive heteroscedasticity. Hence, the entries of the spatial covariance matrix depend not only on the location and the distance between observations, as in nonstationary spatial processes, but also on the variance at nearby locations. Moreover, we illustrate the use of the spARCH model as a residual process for the spatial modelling of lung cancer mortality in U.S. counties.

The remainder of the paper is structured as follows. In Section 2, we introduce the spARCH model and derive important properties of the process. Furthermore, we present some results regarding statistical inference, and we discuss an estimation procedure based on the maximum-likelihood principle. Moreover, the results of various simulation studies are reported to yield better insight into the behaviour of the spARCH process. In Section 4, we demonstrate how our results can be applied in an empirical study. Finally, Section 5 presents the conclusions of the paper.