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A dynamic model for age-specific fertility rates in Italy

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ABSTRACT

Fertility evolution in Italy has shown a deep drop in 1995, and up to now the fertility rate is considered among the lowest in the world. The empirical distribution of the age-specific fertility rates is characterized by a decreasing tendency of the maximum fertility rate and a simultaneous increase of the corresponding mother's age. This tendency has been stimulating recent contributions in modelling and forecasting.

The aim of this paper is to propose a dynamic model for describing and predicting the evolution of the Italian age-specific fertility rates over time. In particular, a well-documented model, such as a Gamma function, slightly modified in order to include time-varying stochastic parameters, is used to describe the systematic and macroscopic variations of the age-specific fertility rates over time, while a nonparametric geostatistical model is applied to describe the correlated residuals at microscopic level. Finally, predictions for the variable under study are provided and main empirical evidences of the temporal evolution for different mother's ages are discussed.

Abbreviations: FR, fertility rates; GARCH, Generalized AutoRegressive Conditional Heteroskedasticity; ARCH, Autoregressive Moving Average; ARIMA, Autoregressive Integrated Moving Average; STRF, spatio-temporal random field; SE, Standard Error; ACF, autocorrelation functions; PACF, partial autocorrelation functions; nls, non-linear least squares regression; MAE, mean absolute error; RMSE, root mean square error.

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1. Introduction

Several contributions on modelling and predicting fertility rates (FR) can be found in the literature. Some of them are related to the Total FR and are based on different approaches, such as GARCH (Generalized AutoRegressive Conditional Heteroskedasticity), ARCH and ARIMA (Autoregressive Integrated Moving Average) time series models (Alders and de Beer, 2004; Keilman and Pham, 2004; Lee and Tuljapurkar, 1994; Lee, 1993; Lee and Wu, 2003), Bayesian models (Raftery et al., 2014) or spatio-temporal geostatistical models (De Iaco et al., 2015). Other models provide excellent fits to the distributions of the age-specific FR referred to specific calendar years (Gayawan et al., 2010; Peristera and Kostaki, 2007) and are useful in social planning (Billari et al., 2012), for both government and private institutions (Hoem et al., 1981; Smith, 1987). Moreover, modern mixture models can capture fertility patterns which exhibit an almost bimodal shape (Chandola et al., 2002). This is common for age-specific FR distributions of developed countries, such as the United Kingdom, Ireland and USA, which are characterized by two populations with different age-specific FR (Azzalini, 1985, 2005; Mazzucco and Scarpa, 2015; Peristera and Kostaki, 2007). Unlike other developed countries, Italy has maintained a classic fertility pattern (a bell shaped distribution, roughly symmetrical though sharper in its left part of its peak) with no additional hump.

In this context, a further step in modelling is related with the possibility of considering that age-specific FR data are often given as a two-way table on a grid, equally spaced in either the vertical (calendar year) and horizontal (mother's age) directions. This type of demographic tables requires not only a temporal analytical perspective, but also a consciousness of the corresponding female age, which in this case can be interpreted as another dimension of the domain. Then, they can be suitably interpreted as data with a spatio-temporal like structure on a two-dimensional domain, i.e. the vertical (time) and horizontal (space) directions. In this case, dynamic fertility tables arise as an alternative to the standard (static) fertility table, with the aim of incorporating the evolution of fertility over time. This is consistent with recent advances in demographic research which are usually based on collection and analysis of individual- and contextual-level data across a wide range of spatial and temporal scales (Matthews and Parker, 2013; Voss, 2007; Weeks, 2004; Débon et al., 2008; Martinez-Ruiz et al., 2010). Booth (2006) offers a wide overview since 1980 about the approaches and developments in demographic forecasting. As remarked by Shang (2015), forecasting methods for age-specific FR can be organized into three classes: parametric (Thompson et al., 1989; Keilman, 2008), semi-parametric (Booth, 1984) and non-parametric models (Bozik and Bell, 1987; Lee, 1993; Hyndman and Shahid Ullah, 2007; Hyndman and Booth, 2008). Nevertheless, none of the forecasting methods in the demographic fertility literature takes into account the existing dependence structure among the data.

In this paper, a new approach in modelling and forecasting age-specific FR is proposed and applied to the Italian FR which has shown a steep decreasing tendency and is considered at the moment among the lowest in the world. The novelty of this paper is (1) to propose an appropriately modified version of an existing parametric model in order to incorporate the macroscopic variation of age-specific rates over time and (2) to treat the correlated residuals at microscopic level (which are usually ignored) by using a two-dimensional geostatistical model.

The rest of the article is organized as follows. Section 2 presents the available two-way table of the Italian age-specific FR for the period 1952–2012. In Section 3 the dynamic model with correlated residuals is introduced and the Italian trend of the age-specific FR (from 13 to 50 years old) over the temporal interval of interest, is reasonably described by a Gamma function (Hoem et al., 1981), with stochastic parameters depending on time. Thus, in Section 3.1 the time-varying stochastic parameters of the Gamma function are estimated and modelled, while in Section 3.2 the age-specific FR correlated residuals (obtained by removing the FR trend component from the observed age-specific FR) are appropriately studied through structural analysis. In Section 4 spatio-temporal kriging based on a suitable class of non-separable covariance models is applied for prediction purposes. Some empirical evidences of the FR evolution in time and for different mother's ages are discussed. Finally, a comparison between the proposed stochastic model with a correlated residual component and the traditional model, commonly used to describe only the systematic structure, is provided in Section 5.