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Statistical analysis of spatially homogeneous dynamic agent-based processes using functional time series analysis

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ABSTRACT

Dynamic systems consisting of multiple interacting autonomous individuals are of particular interest in a number of scientific fields, including ecology, biology, and swarm robotics. Such systems are commonly referred to as agent-based processes. Detection and characterisation of agent–agent interactions is an important step in the analysis of agent-based processes, however existing statistical methods are relatively limited. This paper presents a novel framework for investigating spatial interactions between agents combining techniques from spatial statistics and functional time series analysis. Assuming second order spatial equilibrium of the agent-based process, we develop a test for identifying the specific nature of interactions between agents. We also consider methodology for validating the assumption of spatial equilibrium for a given realisation of the agent-based process. The efficacy of this methodology is demonstrated via Monte Carlo simulation studies and an application to experimental data obtained by observing a species of flightless locust.

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1. Introduction

Dynamic systems of interacting autonomous individuals are encountered in a diverse range of sciences, including ecology, biology, and swarm robotics (Balch, 2000; Plotkin et al., 2000; Cavagna et al., 2008; Buhl et al., 2012; Brambilla et al., 2013; Dale and Fortin, 2014; Johnston et al., 2014; Michalec et al., 2015b; Russell et al., 2015; Binder and Simpson, 2015). Technological developments increasingly allow for the tracking or imaging of individuals in processes arising in these fields, from GPS-tagging of animals (Cagnacci et al., 2010; Dell et al., 2014) and video recordings of animal movements (Cavagna et al., 2008; Buhl et al., 2011, 2012; Michalec et al., 2015a,b) to *in situ* imaging of cells within live animals (Chtanova et al., 2008; Ng et al., 2011; Meijering et al., 2012; Tong et al., 2015). Such systems are typically referred to as agent-based processes and models of these are similarly referred to as agent-based models, or individual-based models.

Observing agent-based processes over time results in sequential sets of positional point data, or point patterns, representing agent positions. For example, an animation of wingless locust nymph movement data, from which two frames are presented in Fig. 1, is presented in the supplementary material. These data are analysed in a later section of this paper. A primary interest in the analysis of such dynamic positional data is to determine the existence and nature of interactions between agents.

While many commonly observed agent-based processes evolve via agent movements, they can also evolve via births and deaths of agents. For example, a population of cells could increase in number via mitosis. Moreover, over longer time scales processes that are not necessarily considered dynamic can in fact be thought of as agent-based processes. For example, the evolution of tree locations in a forest could be considered as a birth–death agent-based process with the probabilities of births and deaths being influenced by the relative position of other trees.

A novel approach used to analyse agent-based models is moment dynamics (Bolker and Pacala, 1997; Law and Dieckmann, 2000; Plank and Law, 2015). Dynamic spatial moments are analogous to the intensity terms in spatial statistics (Diggle, 2003), but describe dynamic agent-based models rather than static point processes. The dynamic first and second spatial moments of an agent-based process can be combined to produce a dynamic pair correlation function and related K function. These functions indicate ranges and types of spatial interaction between agents. Moment dynamic analysis is becoming increasingly important and widespread in the analysis of agent-based models (Raghib et al., 2011; Plank and Law, 2015; Binny et al., 2015).

The study of agent-based processes from a statistical perspective has been limited. Although approaches for the analysis of agent-based processes with respect to pair correlation functions have recently been developed (Cavagna et al., 2008; Binder and Simpson, 2013) these methods have several limitations. Primary among these is the lack of a statistical test of significance for detecting agent–agent interactions from experimental data. Furthermore, existing approaches lack a means of determining when it is reasonable to pool data across observations of an agent-based process to produce a single estimate for the pair correlation function or K function. If the expected spatial arrangement of agents is time invariant, then such data can be pooled, making estimation more accurate. However, if the expected spatial arrangement is not invariant with time, as might occur if the spatial arrangement at observed times is dependent on initial conditions, or if agent–agent interactions are dependent on an environmental factor that varies with time, then the data should not be pooled.

In this paper, we present a general framework for the analysis of agent-based processes that addresses these issues. The methodology that we propose is based on analysing the functional time series that is generated by calculating empirical K functions for each set of agent positional data. The resulting series of curves often exhibit significant autocorrelation due to the dependence between successive spatial arrangements of the agents. The discipline of functional time series analysis has grown considerably in the last decade to provide methodology for the analysis of autocorrelated functions (Ferraty and Vieu, 2006; Hörmann et al., 2010; Horváth and Kokoszka, 2012).

By using techniques in functional time series analysis, we develop a test for stationarity of the empirical K functions. As we will show, the stationarity of empirical K functions suggests that the underlying process is in a type of spatial equilibrium, which we refer to as reduced second moment temporal stationarity. Importantly, if it is determined that an agent-based process exhibits such