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When does the screening effect not hold?

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ABSTRACT

The screening effect is the phenomenon of nearby observations yielding a good approximation to the optimal linear predictor of a spatial process based on a large set of observations. In addition to its obvious relevance to computation with large spatial datasets, knowing when a screening effect occurs is key to understanding the behavior of spatial processes. This work provides the first general results showing when an asymptotic screening effect does not hold by considering the prediction of a weakly stationary, isotropic process on $\mathbb{R}^d$ at a single location based on observing the process everywhere on $\mathbb{R}^d$ with white noise and letting the variance of the white noise tend to 0. The main result shows that a screening effect does not hold if the isotropic spectral density fluctuates too much at high frequencies.

1. Introduction

For a random field Z on $\mathbb{R}^d$ with finite second moments, consider predicting Z at an unobserved location x_0 based on some set of observations $Z(x_1), \ldots, Z(x_n)$ via optimal (minimum mean squared error) linear interpolation, also known as kriging. One might generally expect this optimal linear interpolant to depend mainly on the behavior of Z over distances not much larger than those from x_0 to the nearest observations to x_0. There are a number of ways that one could formalize this concept. One is to compare the mean squared prediction errors of the optimal predictor based on some subset of the observations near x_0 and the optimal predictor based on all of the observations. A closely related formalization is to consider to what extent the behavior of the optimal predictor can be determined just by knowing the behavior of the covariance function in some neighborhood of x_0. Both approaches are used in this paper.

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The phenomenon of obtaining nearly optimal predictors by just using nearest neighbors to x_0 is known as the screening (or screen) effect in the geostatistical literature (Journel and Huijbregts, 1978; Chilès and Delfiner, 2012). Especially for isotropic processes in more than one dimension, exact screening effects, in which distant observations have no effect on the optimal linear predictor, generally do not occur. Thus, it is natural to consider circumstances under which a screening effect holds asymptotically in some limit. Using ϵ to indicate a parameter controlling the observation locations, following the notation in Stein (2011), suppose $x_0 = 0$ and write N_ϵ for some set of observations near 0 and F_ϵ another set of observations generally more distant from 0. Write $e(S)$ for the error of the optimal linear predictor of $Z(0)$ based on observing Z on the set S. Then we will say an asymptotic screening effect holds if

$$\lim_{\epsilon \downarrow 0} \frac{Ee(N_\epsilon \cup F_\epsilon)^2}{Ee(N_\epsilon)^2} = 1. \tag{1}$$

In some settings, it will be convenient to define $N_\epsilon \cup F_\epsilon$ directly and call it U_ϵ. In Stein (2011), $N_\epsilon = \{\epsilon x_1, \ldots \epsilon x_n\}$ and $F_\epsilon = \{y_0 + \epsilon y_1, \ldots, y_0 + \epsilon y_m\}$, where $x_1, \ldots, x_n$ are fixed (not depending on ϵ), distinct, nonzero points in $\mathbb{R}^d$, $y_0 \neq 0$ and $y_1, \ldots, y_m$ are fixed, distinct points in $\mathbb{R}^d$. It is possible to let N_ϵ be a set of fixed size in this setting because the points in F_ϵ are, for small ϵ, all near $y_0 \neq 0$ and thus are well-separated from 0. If we do not want to allow any clear separation between the points in N_ϵ and those in F_ϵ, then we will generally need to allow the number of points in N_ϵ to tend to ∞ as $\epsilon \downarrow 0$ in order for (1) to hold. For example, in Stein (2002), U_ϵ is the infinite lattice $\epsilon(y_0 + j)$ for $j \in \mathbb{Z}^d$, y_0 a fixed point not in $\mathbb{Z}^d$ and N_ϵ is the intersection of this infinite lattice with $r_\epsilon B$, where $r_\epsilon/\epsilon \to \infty$ as $\epsilon \downarrow 0$ and $B \subset \mathbb{R}^d$ contains a neighborhood of the origin. The condition on r_ϵ guarantees that the number of points in N_ϵ tends to infinity as $\epsilon \downarrow 0$. Letting the number of nearby points grow without bound may not be what some have in mind when they think about the screening effect, but for r_ϵ/ϵ bounded, I am unaware of any isotropic model in more than one dimension other than a pure nugget effect for which (1) will hold for this setting.

For a weakly stationary process Z on $\mathbb{R}^d$ with spectral density f,

$$\operatorname{cov}\{Z(x+h), Z(x)\} = \int_{\mathbb{R}^d} e^{i\omega^T h} f(\omega)\, d\omega$$

for all h and x. Stein (2002, 2011) studies conditions on f under which a screening effect holds. For example, Stein (2011) assumes, among other conditions on f, that

$$\lim_{|\omega| \to \infty} \sup_{|\nu| < R} \left| \frac{f(\omega + \nu)}{f(\omega)} - 1 \right| = 0, \tag{2}$$

which says that at high frequencies, the spectral density changes relatively negligibly when the frequency is changed by a modest amount. These works give examples showing that if some constraint is not put on the high frequency behavior of f, then a screening effect may not hold. In these situations, it is often the case that the corresponding autocovariance function possesses some lack of smoothness away from the origin. There are no previous results giving anything like inverses that say if f does not obey a condition such as (2), then a screening effect does not hold.

The rest of this work only considers isotropic processes on $\mathbb{R}^d$, in which case the spectral density $f(\omega)$ depends only on $|\omega|$. Writing $f(u)$ for $u \geq 0$ as this isotropic spectral density, (2) is still applicable with ω and $\omega + \nu$ as positive reals. The symbol $\widehat{\ }$ over a function will be used to indicate a Fourier transform. Specifically, for a measurable function β on $[0, \infty)$ satisfying $\int_0^\infty |\beta(u)| u^{d-1}\, du < \infty$, the function of x on $\mathbb{R}^d$ given by $\int_{\mathbb{R}^d} e^{i\omega^T x} \beta(|\omega|)\, d\omega$ depends only on $r = |x|$ and write $\widehat{\beta}(r)$ for this function on $[0, \infty)$. If, in addition, β is nonnegative, then β is a valid isotropic spectral density for a process on $\mathbb{R}^d$ and $\widehat{\beta}$ is the corresponding isotropic autocovariance function. For $d = 1$, it will be convenient to view β and $\widehat{\beta}$ as even functions on all of $\mathbb{R}$ (e.g., in Section 2) and I will do so without further comment.

The condition (2) has proven useful in distinguishing between models for which a screening effect holds and when it does not. Note that (2) is satisfied by many common models, including all Matérn models (Stein, 1999), all rational spectral densities and a class of spectral densities proposed in Stein (2005, Eq. 4) as a model for space–time processes that allows for arbitrary and different degrees of