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Distances to a point of reference in spatial point patterns

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ABSTRACT

Motivated by a study of social spider behavior, we discuss the distribution of the distances from all the events in a spatial point pattern to a point of reference that has a known location at a given moment of time. The distribution depends on both the shape of the region and the location of the point of reference. The empirical CDF is used to describe the distribution of the distances and compare it to the CDF derived under complete spatial randomness. Empirical distributions are then compared through time focusing on the case in which the point of reference changes with time.

1. Introduction

The interest in the distribution of the distances with respect to a known point of reference emerged while analyzing the spatial location of a mother spider of a subsocial species, *Anelosimus studiosus*, and each one of its offspring in a communal web at a given time. The biological question being examined was whether the spatial distribution of the offspring with respect to the mother changes throughout the day or with the stage of development of the offspring. *A. studiosus* is a subsocial spider in which juveniles share a web with their mother until they mature (Jones and Parker, 2002). They are cooperative foragers, and individual survival depends on the prey capture success of the group.

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When prey is initially snared in the web, the first spider to reach it will slow the prey's movement until other spiders arrive to help subdue it. The mother and her offspring all contribute to building and maintaining the communal web (Jones and Parker, 2000, 2002), resulting in a capture area that is too large to be effectively exploited by a single individual. Thus, it is clear that the spacing of the spiders will affect how quickly a given prey item can be detected and subdued (reducing the chance of the prey escaping). One can imagine that if the spiders are too densely clustered in one area, there would be areas of the web where prey could go undetected. Alternatively, if the spiders are dispersed over too wide of an area, the chance of multiple spiders arriving to assist in cooperative prey capture would be diminished.

Methods from spatial statistics and mathematical modeling have been applied to the experimental data to answer several biological questions (Joyner et al., 2014; Ross, 2013). However, in this article we focus on issues that arise when analyzing the distances from individual juvenile spiders to their mother. This is important in the spider case study, because the mother is the primary forager early in the development of the colony. Nonetheless, the results are applicable to any situation in which one is interested in the distribution of the distances from each of the events in a spatial point pattern to a point of reference.

The distances from the events to a given point, the mother in the case of the spiders, can be studied through its empirical distribution function, which we call $\hat{M}$. The theoretical CDF, M, is derived assuming the event locations are generated under complete spatial randomness (CSR). Some examples of $\hat{M}$ are given in Section 2. M, assuming CSR, is derived in Section 3. It is also of interest to compare $\hat{M}$ for the same colony in two different moments of time on the same day, or at the same time of the day in two different stages of development.

In Section 4 the distribution of the distances to a reference point is considered together with the distribution of the distances to the nearest neighbor (G function) through an example. Clusters might exist but not necessarily around the point of reference. Thus, when there is a point of reference, the M function contributes to the understanding of the process even if the G function has already been applied.

The Kolmogorov–Smirnov test is used in Section 5 to compare the distribution of the distances to the mother for the same colony at the same time of day in two different stages of development in the spider example. Models with repeated measures (the individuals being the colonies) are used to analyze the median distance from the events (offspring) to the point of reference (mother) through time considering all the colonies. The analysis of variance of the models indicates that the location of the mother is useful (small p-value) to explain the median distance from the events to the point of reference.

2. Distances to a given point of reference: the case of the spiders

In a study with spiders in the lab, ten mother spiders were each placed in a different 16 cm × 16 cm enclosure. When the offspring emerged and developed enough as to be individually noticeable, the coordinates of the location of the mother and each of the offspring were recorded 6 times a day (every 4 h) in 4 non-consecutive days (observation day 0, 23, 35 and 48). The mother is not always in the same location. Something of interest in the case of the spiders is the distribution of the offspring with respect to the location of the mother. Fig. 1 displays two relatively extreme situations from our observations. Fig. 1(a) corresponds to Colony A at observation day 0 and Fig. 1(d) corresponds to another colony, Colony B, on observation day 35. Both colonies have a similar number of spiders (25 and 24 respectively in addition to a mother) and observations were done at the same time of day (7 pm). The mother is represented with an icon that resembles a spider and the offspring by dots.

The distances from the events (offspring) to the point of reference (location of the mother, in this example) can be easily calculated. For the examples in Fig. 1(a) and (d), both distributions are skewed with a longer tail to the right for Colony A and to the left for Colony B. Table 1 displays the mean and five-number-summary of the distances of the juvenile spiders to the mother spider in the two examples shown. The spiders in Colony A were in a much earlier stage of development than those in Colony B and tend to be closer to the mother.