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Spatial modelization: Local linear estimation of the conditional distribution for functional data

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ABSTRACT

The main purpose of this paper is to estimate nonparametrically some characteristics of spatial models related to the conditional cumulative distribution of a scalar response given functional random variables by the local linear approach. Specifically, the local linear estimators of the conditional cumulative distribution function and of the successive derivatives of the conditional density are constructed. Then, the asymptotic properties of these estimators, in terms of the almost-complete convergence with rates, are stated. Moreover, the usefulness of our results is illustrated through their application (a) to the spatial conditional mode estimation and (b) to the soil chemistry properties data.

1. Introduction

The study of the conditional distribution, in nonparametric functional statistic, is one of the most important statistical analysis. Such investigation is essentially used to explore and to model the relationship between a response variable and a functional regressor.

In this paper, we consider the problem of the estimation of certain conditional models by using the local linear approach, when the observations are spatially dependent and of functional nature.

Recall first that the nonparametric modelization of functional data has become increasingly popular in the last decades. Key references on this topic can be found in Ferraty and Romain (2011)

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and Ferraty and Vieu (2006). Notice that, first results on the conditional models, in nonparametric functional statistics, concern the asymptotic properties of the kernel estimator. To cite a few, in Ferraty et al. (2006) the almost-complete (a.co.) convergence when estimating the conditional density and its derivatives is established, while the asymptotic normality of the conditional mode estimator is showed in Ezzahrioui and Ould-Saïd (2006) and the L^p-consistency of conditional quantiles estimation is obtained in Dabo-Niang and Laksaci (2012). However, it is well known that the kernel method suffers from some drawbacks, namely, in the bias term. Otherwise, in the nonfunctional case i.e., in the finite dimensional framework, the local polynomial fitting has been recognized to have superior bias properties than the kernel method (cf. Fan and Gijbels, 1996, for more discussions on this subject). Moreover, recently this estimation method has been considered in the statistical data analysis for functional data setup. More precisely, the first results in this direction were obtained in Baìllo and Grané (2009). In the latter paper, it is obtained the L^2-convergence, with rates, of the local linear estimator of the regression function when the explanatory variable takes values in a Hilbert space. Moreover, an alternative definition of this estimator when data take values in a semi-metric space is given in Barrientos-Marin et al. (2010). In this last paper, the almost-complete convergence, with rates, of the proposed estimator is obtained (cf. also El Methni and Rachdi, 2010). Otherwise, concerning the conditional models, several papers (cf. for instance Demongeot et al., 2011a,b, 2010, for more references on this subject) have treated the local polynomial modeling, in terms of the strong estimation of some conditional parameters, when: (a) the explanatory variable takes values in a semi-metric space or is a functional time series, and (b) these observations are both independent and identically distributed (i.i.d.) or dependent (the strongly mixing case).

Then we focus, in this paper, on the extension of investigations on this approach to the local linear modeling when a spatial dependence structure occurs. More precisely, we construct an estimator of the conditional cumulative distribution function, the conditional density and its derivatives by using the fast functional local linear method introduced in Barrientos-Marin et al. (2010). As asymptotic results, we establish the almost-complete convergence, with rates, of the constructed estimator under spatial mixing conditions. As an application of the established results, we derive the same asymptotic results for the conditional mode estimation. This application provides an alternative spatial prediction tool to the classical regression approach. Notice that, we point out that the main difficulties in the analysis of spatial data comes mainly from the fact that points in the N-dimensional space do not have a linear order. So, the classical techniques of time series data analysis cannot be used here. Moreover, we have that the nonparametric modelization of functional spatial data requires some additional tools, that is why we put in place other/new technical procedures permitting to get our results. It should be noticed also that the practical interest of our study comes mainly from the fact that main fields of application of functional statistical methods are related to the analysis of spatial processes indexed by continuous time. Then, it is worth noting that, despite the potential importance of the local linear modeling of spatial data, this subject has not yet been fully explored, as far as we know, only the papers Dabo-Niang et al. (2010), Hallin et al. (2004, 2009) and Xu and Wang (2008), have paid attention to this thematic. On the other hand, to our knowledge just the paper Chouaf and Laksaci (2012) has addressed the functional local linear estimation of the nonparametric spatial regression. By cons, among the lot of papers concerning the nonparametric modelization of spatial and/or functional spatial data we refer to Biau and Cadre (2004), Laksaci and Maref (2009), Dabo-Niang et al. (2010), Dabo-Niang and Thiam (2010), Li and Tran (2009) and Tran (1990), to cite a few and references therein.

This paper is organized as follows. We present the general framework of our study in Section 2. In Section 3, we consider the spatial local linear estimation of the conditional distribution function. The estimation of the conditional density and its successive derivatives are treated in Section 4. Then, while an application on the conditional mode estimation is given in Section 5, Section 6 is devoted to (a) the application of our results to the soil chemistry properties data, and (b) some discussions. Finally, the last section is reserved to the technical proofs of the results of this paper.

2. Presentation of the spatial framework

In order to define the spatial functional version of the local linear estimator of the conditional distribution function, let $Z_{\mathbf{i}} = (X_{\mathbf{i}}, Y_{\mathbf{i}})$ for $\mathbf{i} \in \mathbb{Z}^N$ and $N \geq 1$, be a $\mathcal{F} \times \mathbb{R}$-valued measurable and