Available online at www.sciencedirect.com

SCIENCE DIRECT

Sensors and Actuators A 130–131 (2006) 91–98

SENSORS AND ACTUATORS A PHYSICAL

www.elsevier.com/locate/sna

The constituent equations of piezoelectric multilayer bending actuators in closed analytical form and experimental results

Ruediger G. Ballas [a,*], H.F. Schlaak [a], A.J. Schmid [b]

[a] Darmstadt University of Technology, Institute for Electromechanical Design, Merckstr. 25, D-64283 Darmstadt, Germany

[b] Festo AG & Co. KG, Department Technology & Testing, Ruiter Str. 82, D-73734 Esslingen-Berkheim, Germany

Received 27 May 2005; received in revised form 31 October 2005; accepted 22 November 2005

Available online 24 January 2006

Abstract

In this paper, we present for the first time the derivation of the constituent equations for any kind of clamped-free piezoelectric multilayer bending actuators under different excitation conditions formulated generally for any point over the entire length of the actuator. The constituent equations are presented by a 4×4 matrix $\mathbf{M}$, which combines the extensive parameters mechanical moment M at the end of the bender, force F applied perpendicularly to the tip of the bender, uniform pressure load p applied over the entire length of the bender and applied electrical voltage U with the intensive parameters angular deflection α, deflection ξ, volume displacement V and electric charge Q. In order to verify a part of the derived constituent equations for a clamped-free piezoelectric multilayer bending actuator the bending curvature and force-deflection characteristics of a realized bending actuator are determined experimentally and compared with analytical calculations.

Keywords: Piezoelectric bending actuator; Multilayer; Constituent equations; Closed form solution; Energy concept

1. Introduction

The static constituent equations for a cantilever structure containing two layers, one consisting of a piezoelectric and the second of an elastic material, were described by Smits and Choi [1]. Wang and Cross [2] presented the constitutive equations of symmetrical triple layer piezoelectric benders consisting of two piezoelectric top and bottom layers sandwiched by a non-piezoelectric elastic central layer. DeVoe and Pisano [3] extended the analysis to a multimorph, a cantilever consisting of several layers of dielectric, piezoelectric and elastic materials. Weinberg [4] presented a new analytical solution for piezoelectric bending actuators taking the neutral axis into account which is referred to as the line where the bending strains are zero. Within this analytical concept the dependence of the intensive on the extensive parameters has only been derived for the tip position of piezoelectric bending beam structures. The matrix elements combining the intensive and extensive parameters have not been formulated in a closed form as a function of any point x over the entire length of the beam actuator yet. But just this aspect adds important insights into piezoelectric design.

In this paper the neutral axis, the internal piezoelectric moments and their influences on the bending behaviour of the neutral axis are taken into consideration. In combination with the piezoelectric constitutive equations for the electric displacement and mechanical strain of the ith layer its total energy density w_{tot} can be formulated. The total stored energy W_{tot} of the piezoelectric multilayer actuator is calculated by volume integration of each layer and following summation with respect to the numbers of layers.

Based on the energy concept that has not been used for multilayer bending actuators up till now, the use of the theorem of minimum total potential energy Π is the origin for the calculation of the matrix elements of the 4×4 matrix. In combination with the Rayleigh–Ritz method evaluation functions for the deflection ξ and the generated charge Q for each extensive parameter are defined. With respect to the kinematic boundary conditions of the piezoelectric cantilever and applying the theorem of minimum total potential energy the evaluation functions are determined and the matrix elements can be formulated in a closed form as a function of any point x over the entire length of the beam actuator.

* Corresponding author. Tel.: +49 6151 16 3296; fax: +49 6151 16 4096.
E-mail address: r.ballas@emk.tu-darmstadt.de (R.G. Ballas).

0924-4247/$ – see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.sna.2005.11.034

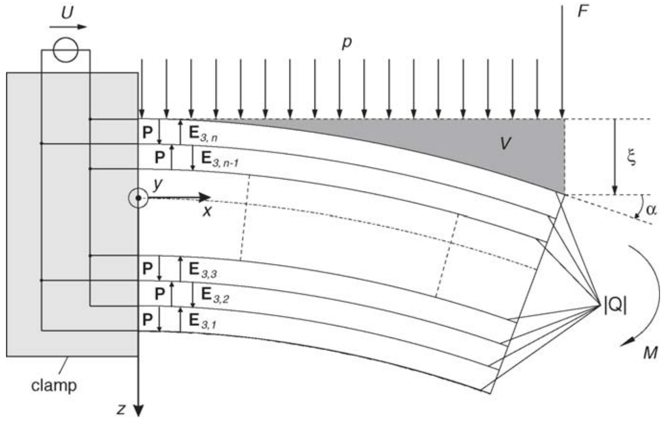

Fig. 1. Nomenclature for the extensive and intensive parameters of a clamped-free piezoelectric multilayer bending actuator.

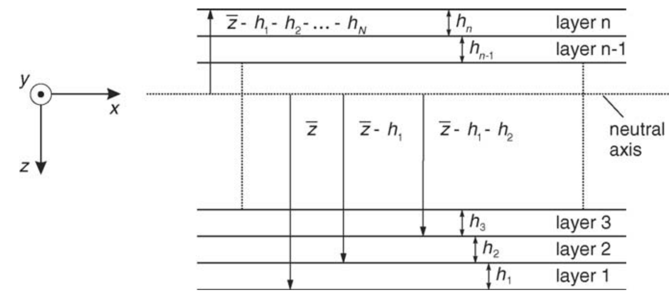

Fig. 2. Definition of the main inertia axis position of a multilayer bending actuator.

2. Assumptions and conventions

The geometry of the piezoelectric multimorph including the intensive and extensive parameters is shown in Fig. 1.

The following assumptions are made:

(a) Each layer consists of either piezoelectric or elastic material.
(b) The interfaces between layers are continuous and do not slip with respect to each other.
(c) The radius of curvature induced by the extensive parameters is much larger than the beam thickness (z-direction).
(d) The beam width (y-direction) is much wider than the thickness, so the beam can be considered in plain strain.

Before deriving the constituent equations for piezoelectric multilayer bending actuators following conventions concerning the crystal axes of the piezoelectric layers are made: the polarization vector **P** defines the 3- or z-direction and the 1- and 2-directions (x- and y-directions) are mutually perpendicular to the 3-direction. Applying an external voltage U results in an electric field $E_{3,i}$ in each layer i. If $E_{3,i}$ is antiparallel (parallel) to the polarization of the piezoelectric element, the material will expand (contract) in the plane perpendicular to $E_{3,i}$ and contract (expand) in the direction of $E_{3,i}$ if $d_{31,i} < 0$ and $d_{33,i} > 0$ that is commonly the case. By implementation of an elastic and passive layer (e.g. layer 1) the beam will bend downwards (upwards) caused by the motional restriction inbetween each layer.

3. Theoretical analysis

3.1. Main inertia axis

Using the Benoulli hypothesis the strain S_1 of a plain beam in x-direction can be described by following equation [5]:

$$S_1(z) = \varepsilon^0 - z\kappa^0, \tag{1}$$

where ε^0 denotes the strain of the neutral axis, κ^0 denotes the curvature, i.e. the bending radius and z is the distance from the main inertia axis. Considering a beam of n-layers (see Fig. 2) being not influenced by external axial forces in x-direction and applying Hooke's law, the stress $T_{1,i}$ in x-direction for the ith layer of the n-layer segment can be described by

$$T_{1,i}(z) = -\frac{z\kappa^0}{s_{11,i}}, \tag{2}$$

where $s_{11,i}$ denotes the compliance of the ith layer.

After bending the n-layer segment by an external moment M each beam element is in static equilibrium resulting in a zero force in x-direction:

$$\sum_{i=1}^{n} F_{1,i} = 0 \tag{3}$$

Substitution of (2) in (3) leads to:

$$\sum_{i=1}^{n} \frac{w_i}{s_{11,i}} \int_{h_{i,\mathrm{u}}}^{h_{i,\mathrm{o}}} z \,\mathrm{d}z = 0. \tag{4}$$

Here $h_{i,\mathrm{u}}$ denotes the lower distance and $h_{i,\mathrm{o}}$ the upper distance of the layer i from the main inertia axis. w_i denotes the width of the ith layer. According to Fig. 2 we get:

$$h_{i,\mathrm{u}} = \bar{z} - \sum_{j=1}^{i} h_j \tag{5}$$

and

$$h_{i,\mathrm{o}} = \bar{z} - \sum_{j=1}^{i-1} h_j \tag{6}$$

Substituting (5) and (6) into (4) and following integration leads to the distance $\bar{z}$ of the main inertia axis from the lower edge of the bending actuator:

$$\bar{z} = -\frac{\sum_{i=1}^{n} \frac{w_i}{s_{11,i}} h_i^2 - 2\sum_{i=1}^{n} \frac{w_i}{s_{11,i}} h_i \sum_{j=1}^{i} h_j}{2\sum_{i=1}^{n} \frac{w_i}{s_{11,i}} h_i} \tag{7}$$

This is the base for the further analytical description.

3.2. Mechanical and piezoelectric moments

The constitutive equations of the piezoelectric effect dependent on the state variables (T_1, E_3) can be reduced from the