Systems & Control Letters 84 (2015) 44–51

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

# Consensus tracking in sensor networks with periodic sensing and switching connected topologies

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ARTICLE INFO

*Article history:*
Received 30 December 2013
Received in revised form
14 April 2015
Accepted 21 April 2015
Available online 24 August 2015

*Keywords:*
Consensus tracking
Multi-agent system
Periodic sensing
Switching topologies

ABSTRACT

This paper studies the consensus tracking of general linear dynamical target in periodically sensing networks with switching and connected sensing topologies. Allowable upper bounds of sensing period are investigated for both time-invariant sensing and time-varying sensing networks. For tracking system with constant sensing period, discretization approach is applied and the tracking problem under switching connected topologies is converted to the robust stability problem of a discrete-time uncertain system. Then an allowable upper bound of sensing period is given by solving optimal $H_\infty$ control problem, and an explicit bound, which is composed of the eigenvalues of target's dynamic matrix and topology matrix, is further provided. For tracking system with time-varying sensing period, input-delay approach is applied and a bound is given by using Lyapunov–Krasovskii functional analysis and solving the feasibility of three LMIs. Numerical examples are given to illustrate the results.



## 1. Introduction

Distributed tracking is one of important applications in sensor networks. The sensors of tracking vehicles measure the sensing field locally. Using the available sensing information, the vehicles control their trajectories cooperatively, with the goals of tracking the target.

One distributed approach used in tracking control is built based on consensus, which means that a group of agents reach an agreement on a common value. Consensus has been widely studied for systems with first-order, second-order, and high-order dynamical agents. When tracking a constant leader (target) state, first-order consensus strategy can be applied. Static consensus has been widely studied for systems with switching interaction topologies [1], delays [2], sampled control [3], noise [4], etc. Consensus with a dynamic leader is comparatively complicated. Existing results mainly focus on dynamic leader with integrator dynamics. When the leader's acceleration input is available to all followers, Hong et al. [5] analyzed consensus tracking algorithm under switching undirected topology, Hong et al. [6] and Peng et al. [7] proposed neighbor-based rules for every agent to track a leader whose states may be not measured and analyzed the consensus problem for system with switching topologies and time-varying delays. When the estimates of the leader's velocity and the followers' velocities are available to each agent, Cao and Ren [8,9] proposed a proportional-and-derivative-like consensus tracking algorithm in both continuous-time and discrete-time settings. For dynamic target with general linear time-invariant (LTI) dynamics, tracking and formation control was addressed for a team of autonomous agents that evolve dynamically in a space containing a measurable vector field in [10]. The decentralized estimation strategy was developed and consensus was analyzed for systems under periodically switched communication with sufficiently fast switching rate. Other consensus analysis for general LTI multi-agent systems can be found in [11–14].

Sensors work periodically and the sensing period may be time-varying in order to save the power of sensors and improve the system performance or varies with the network condition. There have been some discussions for consensus of sampled-data multi-agent systems with sampling period. For systems with constant sampling period, discretization is a commonly used approach [15–18]. Cao and Ren [15] studied two sampled-data-based discrete-time coordination algorithms for multi-vehicle systems with double-integrator dynamics under dynamic directed interaction. Sufficient conditions on the interaction graph, the damping gain and the sampling period to guarantee coordination were proposed by using the property of infinity products of stochastic

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http://dx.doi.org/10.1016/j.sysconle.2015.04.010
0167-6911/

matrices. Chen and Li [16] investigated observer-based consensus of second-order multi-agent systems with fixed and stochastically switching topology via sampled data, and gave sufficient and necessary conditions of consensus on parameters and sampling period. Zhang and Tian [17] studied consensus of sampled-data second-order multi-agent systems with packet loss and delays, and gave consensus conditions. Katayama [18] considered sampled-data consensus control for nonlinear multi-agent systems of strict-feedback form. Based on the hybrid system analysis, Zhang and Tian [19] studied general linear sampled-data multi-agent systems and gave an allowable upper bound of sampling period for system under given protocols. By converting the sampled-data system to the discrete-time system with delays, Gao and Wang [20] studied the consensus of second-order multi-agent systems where the sampling period of each agent was independent of the others' and the topology was varying. They pointed out that if the union graph of all direct graphs had a spanning tree, then there existed controller gains and sampling periods such that consensus was reached. For second-order multi-agent systems with asynchronous sampling, Gao and Wang [21] transformed the consensus problem to the global asymptotic stability problem of a continuous-time switched system with time-varying delays and proposed admissible upper bound of delays based on LMIs. Wen et al. [22] studied the consensus of multiple agents with intrinsic nonlinear dynamics and sampled-data information. By converting the sampled-data multi-agent system to an equivalent nonlinear system with a time-varying delay, the upper bound of the maximal allowable sampling intervals was obtained via solving LMIs.

In this paper we study tracking problem of general LTI dynamical target in sensor networks. The sensors are homogeneous and work periodically, the topology is switching and keeps connected. Allowable upper bound of sensing period is focused on. As long as the sensing period is less than this bound, there exist linear consensus tracking controllers driving the followers tracking the target. When the sensing period is time invariant, we first perform a series of system transformations and convert the consensus tracking problem to the robust stability of certain uncertain discrete-time system with the uncertainty corresponding to the topologies. Then from $H_\infty$ stability analysis and solving the optimization problem, an LMI-based bound is obtained. Moreover an explicit bound for sensing period, which is composed of the eigenvalues of leader's dynamic matrix and Laplacian matrix, is further provided based on the fact that the optimal $H_\infty$ norm is less than the multiplication of system matrix's unstable eigenvalues. When the sensing period is time-varying, through delayed-input approach we convert the consensus tracking problem to the robust stability of an uncertain delayed system. Then from Lyapunov–Krasovskii functional analysis, we obtain a tracking condition in form of three LMIs. By solving the robust stabilization problem an allowable sensing period bound is proposed. Whether the sensing period is time invariant or time-varying, the computation of the proposed bound in this paper is not increasing with the number of tracking vehicles and switching topologies.

This paper is organized as follows. Section 2 describes the target dynamics, sensors' sensing model, and consensus tracking algorithm. Section 3 investigates the allowable period bound for the network with time invariant sensing period, while Section 4 focuses on the allowable period bound for the network with time-varying sensing period. Finally, Section 5 presents simulation examples to validate the results.

## 2. Problem formulation

Consider a target with continuous-time linear dynamics

$$\dot{x}_0 = Ax_0(t) \tag{1}$$

where $x_0 \in R^p$ denotes the state of tracking target, the system matrix $A$ is not Hurwitz stable.

Suppose there are $n$ vehicles with the same type of sensor in the team. The sensing model is

$$y_{ij}(t) = Hx_j(t) \tag{2}$$

where $y_{ij} \in R^q$ is the observation of $j$ made by the sensor of vehicle $i$, $j = 0, 1, \dots, n$, $i = 1, 2, \dots, n$. $(A, H)$ is completely observable.

The objective is to perform distributed tracking for a target that evolves according to (1). We apply consensus tracking algorithm as following

$$\begin{aligned}\dot{x}_i &= Ax_i(t) + b_i(t)K(y_{i0}(t) - Hx_i(t)) \\ &\quad + K\sum_{j=1}^{n} a_{ij}(t)(y_{ij}(t) - Hx_i(t))\end{aligned} \tag{3}$$

where if the target is in the sensing range of vehicle $i$, then $b_i(t) > 0$, otherwise $b_i(t) = 0$. If vehicle $j$ is in the sensing range of $i$, then $a_{ij}(t) > 0$, otherwise $a_{ij}(t) = 0$. Obviously $a_{ij}(t) = a_{ji}(t)$ and thus the topology among followers is undirected. The nonzero weights $a_{ij}$ and $b_i$ are given. $N_i(t) = \{j : a_{ij}(t) > 0\}$. $K$ is the control gain to be designed.

Since the target and the tracing vehicles are moving, the sensing topology among them may be dynamically changing. Here we assume there are $N$ possible sensing topologies. Denote the set of all possible topologies by $\{\mathcal{G}_1, \dots, \mathcal{G}_N\}$, the corresponding Laplacian matrices among the followers by $L_i$, $i = 1, \dots, N$, and the corresponding link matrix between the target and followers by $B_i$, $i = 1, \dots, N$, then $L(t) + B(t) \in \{L_i + B_i,\ i = 1, \dots, N\}$. It is assumed in this paper that each topology $\mathcal{G}_i$ is connected and at each instant there is at least one vehicle detecting the target node.

Suppose all vehicles' sensors are clock synchronized and each sensor works periodically. During each sensing period, the vehicle uses the available sensing information to update its state. Denote $t_k$ as the sensing instant, thus for $t_k \le t < t_{k+1}$, $k = 0, 1, \dots,$

$$\begin{aligned}\dot{x}_i &= Ax_i(t) + b_i(t_k)K(y_{i0}(t_k) - Hx_i(t_k)) \\ &\quad + K\sum_{j=1}^{n} a_{ij}(t_k)(y_{ij}(t_k) - Hx_i(t_k)).\end{aligned} \tag{4}$$

Denote $e_i = x_i - x_0$ as the tracking error of vehicle $i$, $i = 1, \dots, n$. From (1), (4), for $t \in [t_k, t_{k+1})$ the tracking errors evolve according to

$$\begin{aligned}\dot{e}_i &= Ae_i(t) - b_i(t_k)KHe_i(t_k) \\ &\quad + KH\sum_{j=1}^{n} a_{ij}(t_k)(e_j(t_k) - e_i(t_k)).\end{aligned} \tag{5}$$

Define the aggregate vectors of tracking errors $e(t) = [e_1^T(t), e_2^T(t), \dots, e_n^T(t)]^T$, diagonal matrix $B = \mathrm{diag}\{b_1, \dots, b_n\}$, and Laplacian matrix $L = [l_{ij}]_{n\times n}$ with $l_{ij} = -a_{ij}$ when $i \ne j$, $l_{ii} = \sum_{j=1}^{n} a_{ij}$. Then the dynamics (5) can be compactly written as

$$\dot{e} = (I_n \otimes A)e(t) - ((B(t_k) + L(t_k)) \otimes KH)e(t_k) \tag{6}$$

where $I_n$ is identity matrix with dimension $n$, $\otimes$ denotes the Kronecker product of two matrices.

The vehicles track the target asymptotically if and only if the tracking error $e(t)$ converges to zero asymptotically. If there exists control gain $K$ such that the vehicles track the target asymptotically, we say the consensus tracking problem is solvable using tracking algorithm (3).