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Observer design for a class of nonlinear ODE–PDE cascade systems

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ABSTRACT

The problem of state-observation is addressed for nonlinear systems that can be modeled by an ODE–PDE series association. The ODE subsystem assumes a triangular structure while the PDE element is of heat diffusion type. The aim is to accurately estimate online the state vector of the ODE subsystem and the distributed state of the PDE element. One major difficulty is that the state observation must only rely on the global system output i.e. the PDE state at the terminal boundary. In particular, the connection point between the ODE and the PDE blocks is not accessible to measurements. The observation problem is dealt with by designing a high-gain type observer. Sufficient conditions involving the PDE domain length are formally established that ensure the observer exponential convergence.

1. Introduction

In the last decades, the problems of nonlinear system observability and observer design have intensively been investigated for systems that can be described by ordinaries differential equations (ODEs). Several types of observers have been proposed, for several classes of nonlinear systems, including the high-gain observer e.g. [1–4], sliding-mode observers e.g. [5–7], Luenberger-like observers e.g. [8]. Additional references can be found in recent monographs e.g. [9,10].

The problem of infinite dimensional system (IDS) observability and observer design has also been given a great deal of interest, especially in recent years. The earliest works have focused on linear IDSs and a relatively complete theoretical framework exists since the nineties, including the infinite dimensional Luenberger observer, e.g. [11,12] and reference list therein. Boundary observer design of bilinear IDSs has been studied in e.g. [13–15]. A unified study of both interior and boundary observation for linear and bilinear systems is found in [16]. In [17], backstepping techniques have been used to design exponentially convergent boundary observers for a class of parabolic partial integro-differential equations. The problem of initial state recovery has also been given interest. In [18], an iterative algorithm is proposed to recover the initial state of a linear infinite dimensional system. The proposed algorithm generalizes various algorithms, proposed earlier for specific classes of systems, and stands as an alternative to methods based on Gramian inversion [19]. The ideas of [18] have been extended to some nonlinear infinite dimensional systems, using LMI techniques [20].

In this paper, we are interested in state observation of cascade systems including an ODE subsystem followed in series with PDE subsystem (Fig. 1). The aim is to recover the (finite-dimension) state of the ODE part and the (infinite-dimension) state of the PDE part. One major difficulty of this problem lies in the fact that the connecting point between the two parts is not accessible to measurements. In [21,22] a boundary observer has been developed for a cascade involving a linear ODE and a (linear) heat PDE equation that may represent a distributed state sensor. In turn, the observer assumes a cascade structure with a finite- and infinite-dimensional parts. The observer design relies upon an infinite-dimensional transformation, inspired from the backstepping principle, and an exponentially stable target system. The observer thus obtained is shown to be exponentially convergent in the sense of a quadratic norm. Inspired by [21,22], a new observer design is presently developed to address ODE–PDE systems that involve a triangular nonlinear ODE subsystem (the PDE part remains a heat equation).

The novelty of the present design approach is twofold: (i) it combines the backstepping infinite-dimensional transformation of [22] and the high-gain observer design principles [1,3,4]; (ii) it involves a quite different target system (as the ones used in [22] are not usable for the present problem).

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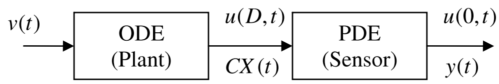

Fig. 1. System structure.

The paper is organized as follows: first, the observation problem under study is formulated in Section 2; then, the observer design and analysis are dealt with in Section 3; a conclusion and reference list end the paper. To alleviate the presentation, some technical proofs are appended.

Notations. Throughout the paper, $\mathbf{R}^n$ denotes the n dimensional real space and the corresponding Euclidean norm is denoted $\|.\|$. $\mathbf{R}^{n\times n}$ denotes the set of all $n \times m$ real matrices and $\|.\|$ the induced Euclidean norm. Functions that are continuously differentiable with respect to all their arguments are denoted C^1. $L_2[0,\ D]$ is the Hilbert space of square integrable functions and the corresponding L_2 norm is denoted $\|.\|_2$. Accordingly, $\|\eta\|_2 = \left(\int_0^D \eta^2(\varsigma)d\varsigma\right)^{1/2}$ for all $\eta \in L_2[0,\ D]$. $H^1(0, D)$ is the Sobolev space of absolutely continuous functions $\eta : [0, D] \to \mathbf{R}$ with $d\eta/d\varsigma \in L_2[0, D]$. $H^2(0, D)$ is the Sobolev space of scalar functions $\eta : [0, D] \to \mathbf{R}$ with absolutely continuous $d\eta/d\varsigma \in L_2[0, D]$ and $d^2\eta/d\varsigma^2 \in L_2[0, D]$.

2. Problem formulation

Analytically, the system under study is modeled by a finite-order nonlinear ODE connected in series with a PDE (Fig. 1). The former could represent the plant dynamics which presently assume the following triangular-form state-space representation:

$$\dot{X}(t) = AX(t) + Bv(t) + f(X(t)), \quad t \geq 0 \tag{1a}$$

$$u(D, t) = CX(t) \tag{1b}$$

with:

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ \vdots & & & & \ddots & 1 \\ 0 & 0 & \cdots & \cdots & \cdots & 0 \end{pmatrix} \in \mathbf{R}^{n\times n}, \tag{1c}$$

$$B \in \mathbf{R}^n, \qquad C = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} \in \mathbf{R}^{1\times n}$$

where $v \in C([0, \infty) : \mathbf{R})$ denotes the system input, $X \in \mathbf{R}^n$ the state vector and $f : \mathbf{R}^n \to \mathbf{R}^n$ is a vector field with the triangular form:

$$f(X) = \begin{pmatrix} f_1(X_1) \\ f_2(X_1, X_2) \\ \vdots \\ f_n(X_1, \ldots, X_n) \end{pmatrix}; \quad f_i : \mathbf{R}^i \to \mathbf{R}. \tag{1d}$$

It is supposed that $f(0) = 0$ and f is class C^2 with bounded Jacobian matrix i.e.

$$\exists \beta > 0, \quad \forall X \in \mathbf{R}^n : \ \|f_X(X)\| \leq \beta. \tag{1e}$$

The system PDE part represents a diffusive sensing system modeled by the following heat equation and associated boundary condition:

$$u_t(x, t) = u_{xx}(x, t), \quad 0 \leq x \leq D \tag{2a}$$

$$u_x(0, t) = 0, \qquad u(D, t) = CX(t) \tag{2b}$$

where D is a known scalar representing the length of the PDE domain. The whole system is observed through the output signal,

$$y(t) \overset{def}{=} u(0, t). \tag{2c}$$

The aim is to design an observer that provides accurate online estimates of both the finite-dimension state vector $X(t)$ and the distributed state variable $u(x, t)$, $0 \leq x \leq D$. The observer must only make use of the system input $v(t)$ and output $y(t)$. In particular, the connection signal $u(D, t)$ is not supposed to be accessible to measurements.

Note that, a similar state observation problem has been dealt with in [22] for ODE–PDE systems where the ODE subsystem is linear i.e. the vector field $f(.)$ is identically null.

Before proceeding with the observer design and analysis, let us check that the system described by (1a)–(1e)–(2a)–(2c) is well posed. This is the subject of following statement proved in Appendix A.

Proposition 1. *The system (1a)–(1e)–(2a)–(2c) has a unique classical solution*

$$u(t) \in C([0, \infty) : Y) \cap C^1((0, \infty) : Y),$$
$$X(t) \in C^1([0, \infty) : \mathbf{R}^n)$$

provided that $u(0) \in Y$, *with* $Y = \{\xi \in H^2(0, D) : \ \xi(D) = 0, \xi_x(0) = 0\}$. □

3. Observer design and analysis

3.1. Observer design

Inspired by the high-gain observer design approach, the following observer structure is considered for the system (1a)–(1d)–(2a)–(2c):

$$\dot{\hat{X}} = A\hat{X} + Bv(t) + f(\hat{X}) - \theta\Delta^{-1}K(\hat{u}(0, t) - y(t)) \tag{3a}$$

$$\hat{u}(D, t) = C\hat{X}(t) \tag{3b}$$

$$\hat{u}_t(x, t) = \hat{u}_{xx}(x, t) - \theta k(x)(\hat{u}(0, t) - y(t)) \tag{3c}$$

$$\hat{u}_x(0, t) = 0 \tag{3d}$$

for all $t \geq 0$ and all $x \in [0,\ D]$, with

$$\Delta \overset{def}{=} diag\left\{1, \frac{1}{\theta}, \ldots, \frac{1}{\theta^{n-1}}\right\} \in \mathbf{R}^{n\times n} \tag{3e}$$

where the scalar $\theta > 1$ is a design parameter. The vector and scalar gains, $K \in \mathbf{R}^n$ and $k(x) \in \mathbf{R}$, have yet to be defined. To this end, introduce the state estimation errors:

$$\tilde{X} = \hat{X} - X, \qquad \tilde{u} = \hat{u} - u. \tag{4}$$

Then, subtracting each of the system equations (1a)–(1b)–(2a)–(2c) from the corresponding equation in the observer (3a)–(3d), one gets the following error system:

$$\dot{\tilde{X}} = A\tilde{X} + \left(f(\hat{X}) - f(X)\right) - \theta\Delta^{-1}K\,\tilde{u}(0, t) \tag{5a}$$

$$\tilde{u}(D, t) = C\tilde{X}(t) \tag{5b}$$

$$\tilde{u}_t(x, t) = \tilde{u}_{xx}(x, t) - \theta k(x)\,\tilde{u}(0, t) \tag{5c}$$

$$\tilde{u}_x(0, t) = 0. \tag{5d}$$

Inspired by [22], the following backstepping transformation is considered:

$$\tilde{Z} = M^{-1}(D)\Delta\tilde{X}, \tag{6a}$$

$$\tilde{w}(x, t) = \tilde{u}(x, t) - CM(x)\tilde{Z}(t) \tag{6b}$$

where $M(x)$ is a matrix function yet to be defined. Then, the error system (5a)–(5d) rewrites, in terms of $\tilde{Z}$ and $\tilde{w}(x, t)$, as follows (see Appendix B):