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On the inversion of a class of nonlinear systems

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ABSTRACT

This paper deals with the inversion problem of affine square multi-input multi-output (MIMO) nonlinear systems. It presents a new algorithm unifying the construction of the inverse of a dynamical system with a regular or a singular characteristic matrix. This algorithm is based on the determination of a projector on the fibre bundle of the state space and incorporates a regularization of the singular case. It has the advantage of avoiding the input derivatives. Numerical examples are given to illustrate the proposed approach.

1. Introduction

The problem of inversion dynamics and invertibility of dynamical systems has been a subject of a great deal of research since the works of [1,2] for linear dynamical systems. Then it has been extended to nonlinear dynamical systems in [3–10]. In fact, inversion dynamics problem is of utmost importance to resolve issues associated with the control field. It arises in the problem of tracking a reference trajectory, in the robot inverse kinematics and it occurs in the inversion of the observability map of nonlinear dynamical systems. Broadly speaking, the inverse dynamics involves a decomposition of an input–output dynamical system into an external part, that enables an explicit relationship between inputs and outputs, and an internal part governed by dynamics that involve only internal state that does not bring into play inputs. These last dynamics provide the so-called zero dynamics when the external variables are kept to zero. For single input single output (SISO) linear dynamical systems the inverse dynamics has been fully characterized by their transfer functions [2]. The same problem also was solved in [4] for SISO nonlinear case, where a full-order realization was given.

However, in the case of MIMO nonlinear dynamical systems, the inversion dynamics problem is rather difficult to solve. Several researchers have dealt with this problem. In [11] the concept of zero dynamics was connected to the inverse dynamics. Then, [12] provided a nice interpretation of input–output linearization via a feedback removing the zero dynamics.

Despite intensive research on this subject, there only are few of them that provide computational algorithms of the inverse dynamics: [3] gave an algorithm to compute the inverse and the zero dynamics (see also [13]).

Another alternative method to address this problem is to determine the class of nonlinear dynamical systems which are input–output linearizable. Necessary and sufficient geometric conditions have been stated in [14] and [15].

Therefore, the main priority objective of this paper is to supply a new algorithm to compute the inverse dynamics for affine MIMO nonlinear control systems with regular characteristic matrix as well for singular one. In this last case, it provides another algorithm to increase the rank of the singular characteristic matrix. Indeed, when a given characteristic matrix has no full rank, then the algorithm provides new outputs to increase its rank. Moreover, the proposed algorithm has the advantage of not requiring the input derivatives. Therefore the smoothness of inputs is not needed.

This paper is organized as follows: Section 2 summarizes necessary and sufficient geometrical conditions for solvability of the inverse dynamics problem for nonlinear MIMO systems with regular characteristic matrix. Moreover, it recalls the involutivity concept from the differential forms approach. Section 3 presents singular characteristic matrix case, first it provides an algorithm to increase

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the rank of the singular characteristic matrix and then, states sufficient conditions to solve the inverse dynamics problem. Section 4 provides a geometrical algorithm using a projector to compute the inverse dynamics.

2. Notations and inverse dynamics for regular characteristic matrix

This section provides a basic summary on necessary and sufficient conditions for the existence of the inverse dynamics for a nonlinear system with a regular characteristic matrix. To achieve this goal, we consider the following MIMO dynamical system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in U \subseteq \mathbb{R}^n$ represents the state, $y = (y_1, \dots, y_m)^T \in \mathbb{R}^n$ represents the output and $u = (u_1, \dots, u_m)^T \in \mathbb{R}^m$ is the vector of the inputs, perturbations or faults.

Let us introduce an informal definition for the invertibility of a dynamical system.

Definition 1. Dynamical system (1)–(2) is said to be invertible if we can reconstruct its inputs from its outputs, their derivatives and the internal state of the system.

The realization of inverse dynamics of dynamical system (1)–(2) is given by an auxiliary dynamical system described by (see [16]).

$$\dot{\eta} = \varphi\left(\eta, y, \dot{y}, \dots, y^{(p)}\right) \tag{3}$$

$$u = \omega\left(\eta, y, \dot{y}, \dots, y^{(q)}\right) \tag{4}$$

where p and q are vectors of integers depending on the well-known relative degrees and will be given hereafter. The variable state η is the so-called internal state. It represents the part of state which is not linked to inputs. Its determination is particularly crucial to describe the inverse dynamics. To the best of our knowledge, in the available literature there is no constructive computational algorithm to determine η.

Before stating the existence conditions of the inverse dynamics for (1)–(2), we revisit some basic elements from differential geometry required within this framework.

Background on some geometrical material

- A vector field can be considered as a derivation operator $f = \sum_{i=1}^{n} f_i(x)\frac{\partial}{\partial x_i}$ where f_i for $i =: n$ is function of state x.
- The Lie derivative of a function $h(x)$ in the direction of f is itself a function given by $L_f h = \sum_{i=1}^{n} f_i(x)\frac{\partial h}{\partial x_i}(x)$.
- If $g = \sum_{i=1}^{n} g_i(x)\frac{\partial}{\partial x_i}$ is another vector field then, the Lie bracket of f and g is itself a vector field given by $[f, g] = \sum_{i=1}^{n}\left(L_f g_i(x) - L_g f_i\right)\frac{\partial}{\partial x_i}$.
- A differential 1-form ω on an open set $U \subseteq \mathbb{R}^n$ endowed with coordinates $x = (x_1, \dots, x_n)$ is given by $\omega = \sum_{i=1}^{n}\kappa_i(x)dx_i$. Its evaluation on a vector field f is a function given by $\omega(f) = \sum_{i=1}^{n} f_i(x)\kappa_i(x)$.

Within this paper we will assume the following.

Assumption 1. 1. The distribution $\Delta = Span\{g_1, \dots, g_m\}$ has dimension m and is involutive i.e. $[g_i, g_j] \in \Delta$ for all $1 \le i, j, \le n$. We will say that Δ is closed under Lie bracket.

2. There exist integers $r_1, \dots, r_m$, such that for $1 \le i \le m$, $\exists k_i \in \{1, \dots, m\}$ such that

$$dL_f^{j-1}h_i(g_s) = 0, \quad \text{for all } s \neq k_i,\ 1 \le j < r_i$$
$$dL_f^{r_i-1}h_i(g_{k_i}) \neq 0$$

where for $i \in \{1, \dots, m\}$ the function $L_f^{r_i-1}h_i$ is the $(r_i - 1)$th Lie derivative of h_i in the direction of the vector field f. Its differential is a 1-form and it can be evaluated on a vector field here g_{k_i}. The numbers $r_1, \dots, r_m$ are the so-called relative degrees [13] and $r = r_1 + \cdots + r_m \le n$ is the total relative degree.

3. The characteristic matrix of the dynamical system (1)–(2)

$$\Gamma(x) = \begin{pmatrix} L_{g_1}L_f^{(r_1-1)}h_1 & \dots & L_{g_m}L_f^{(r_1-1)}h_1 \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ L_{g_1}L_f^{(r_m-1)}h_m & \dots & L_{g_m}L_f^{(r_m-1)}h_m \end{pmatrix}$$

is regular i.e. it has a full rank m.

Now, consider the following partial change of coordinates

$$\xi_i = \left(\xi_{i,1}, \xi_{i,2}, \dots, \xi_{i,r_i}\right)^T = \left(h_i, L_f h_i, \dots, L_f^{(r_i-1)}h_i\right)^T. \tag{5}$$

Then we can state the following theorem (see [13]).

Theorem 1. *Under condition 2 of Assumption 1 there locally exist $(n - r)$ variables $\eta = (\eta_1, \dots, \eta_{n-r})$ independent of ξ such that in coordinates (ξ, η) the system (1)–(2) can be rewritten as follows:*

$$\dot{\xi}_{i,j} = \xi_{i,j+1} \quad \text{for } 1 \le i \le m \text{ and } 1 \le j \le r_i - 1 \tag{6}$$

$$\dot{\xi}_{i,r_i} = b_i(\xi, \eta) + \sum_{j=1}^{m} a_{i,j}(\xi, \eta)u_j \quad \text{for } 1 \le i \le m \tag{7}$$

$$\dot{\eta} = \bar{f}(\xi, \eta) \tag{8}$$

$$y_i = \xi_{i,1} \tag{9}$$

where $b_i(\xi, \eta) = L_f^{r_i}h_i$ for $i = 1 : m$ and $a_{i,j} = L_{g_j}L_f^{r_i-1}h_i$ for $j = 1 : m$ are the $\Gamma(x)$ coefficients.

Therefore, from Eqs. (6)–(8), we can deduce the following inverse dynamics:

$$\begin{cases} \dot{\eta} = \bar{f}(\xi, \eta) \\ u = \Gamma^{-1}(\xi, \eta)\left(\begin{pmatrix} \dot{\xi}_{1,r_1} \\ \dot{\xi}_{2,r_2} \\ \dots \\ \dot{\xi}_{m,r_m} \end{pmatrix} - \begin{pmatrix} b_1(\xi, \eta) \\ b_2(\xi, \eta) \\ \dots \\ b_m(\xi, \eta) \end{pmatrix}\right). \end{cases} \tag{10}$$

From definition (5) of ξ and Eqs. (6)–(9) it can be seen that $y_i = \xi_{i,1}$ is the first coordinate of each subsystem (6) for $i \in \{1, \dots, m\}$. Therefore, from (6) we have $\xi_{i,j+1} = \dot{\xi}_{i,j} = y_i^{(j)}$ the jth derivative of the output y_i for $j \in \{2, \dots, r_j - 1\}$. Then $\xi_i = (y_i, \dot{y}_i, \dots, y^{(r_i-1)})$ for $i = 1 : m$. Thus, (10) is in the form (3)–(4) with $p = (r_1 - 1, \dots, r_m - 1)$ and $q = (r_1, \dots, r_m)$. More specifically, we have

$$\begin{cases} \dot{\eta} = \bar{f}(\xi, \eta) \\ u = \Gamma^{-1}(\xi, \eta)\left(\begin{pmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \\ \dots \\ y_{r_m}^{(r_m)} \end{pmatrix} - \begin{pmatrix} b_1(\xi, \eta) \\ b_2(\xi, \eta) \\ \dots \\ b_m(\xi, \eta) \end{pmatrix}\right). \end{cases} \tag{11}$$

Remark 1. Let $\Delta^\perp$ be the co-distribution annihilator of Δ. As Δ is of rank m and involutive, $\Delta^\perp$ is of rank $(n - m)$. Therefore, by Frobenius's theorem $\Delta^\perp$ is locally spanned by