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Distributed output tracking of high-order nonlinear multi-agent systems with unstable linearization☆

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ABSTRACT

This paper addresses the problem of distributed output tracking for a class of high-order nonlinear multi-agent systems whose linearization parts may have unstable modes. For the case where the graph topology is directed and the leader is the neighbor of only a small portion of followers, distributed tracking control laws are designed. By using the algebra graph theory, it is shown that all the states of the closed-loop system are bounded, and the tracking errors can be tuned to be arbitrarily small. Finally, the design procedure is applied to underactuated unstable mechanical multi-agent systems, from which the efficiency of the tracking controllers is demonstrated.

1. Introduction

In the past decade, cooperative control of the network of dynamic agents has attracted considerable attention of researchers because of the increasing interest in its broad applications in many areas including formation control of unmanned air vehicles [1], attitude alignment of clusters of satellites and load balancing [2]. Leader-following distributed tracking control is concerned with multi-agent systems (MASs), where the leader is a particular agent whose motion is independent of the other agents and is to be followed by the other agents. Therefore, one can realize control objectives by only controlling the leader, which converts the control problem of the multi-agent systems into that of a single agent. Specifically, the leader-following tracking problems for various MASs, such as single-integrator systems [3], double-integrator systems [4] and general linear systems [5], have been extensively studied. When stochastic noise exists in the communication channel, [6] constructs a common Lyapunov function to make each agent track the active leader; [7] gives a sufficient condition to ensure strong mean square consensus under the fixed topologies; [8] proves that the first-order followers' states can track the leader's states in the mean square sense by estimating the velocity of the active leader; [9] investigates the sampled-data based consensus tracking problem of first-order multi-agent systems.

In comparison with the development of networked linear systems, there is very few results available about nonlinear MASs control due to the complex of the system structure. [10] studies first-order nonlinear systems with strongly connected graphs. [11,12] investigate the leader-following consensus problem of second-order nonlinear MASs with global Lipschitz assumption. [13] employs the adaptive control technique to deal with the uncertainties appeared in multiple uncertain mechanical systems. [14] investigates the cooperative tracking control of higher-order nonlinear systems with Brunovsky form. [15] studies the distributed robust cooperative tracking problem for multiple non-identical second-order nonlinear systems with bounded external disturbances. It is worthwhile to mention that in these studies, all the models considered are weakly nonlinear or based on the assumptions that the Jacobian linearizations of nonlinear systems are stabilizable and detectable. However, when the nonlinear MASs whose Jacobian linearization may have unstable modes associated with eigenvalues on the right-half plane, to the best of our knowledge, there is no any results about the distributed output tracking control about such systems.

In this paper, we consider the problem of distributed output tracking for a class of high-order nonlinear MASs with unstable linearization under a directed graph topology. Compared with the available results, the main contributions of this paper are as follows:

(1) The nonlinear MASs considered in this paper are with high-order and unstable modes, which include many multi-agent

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models as special cases, such as linearizable nonlinear MASs [11] or weakly nonlinear MASs [14].

(2) The systems considered in [16,17] are single-agent nonlinear systems with unstable linearization whose output tracking problem can be handled by adding a power integrator method as in [18] since these systems are single-input and single-output. In contrast, for MASs, the closed-loop system is multi-input and multi-output and thus we have to develop other specific techniques to deal with the distributed output tracking problem since the existing method is invalid.

(3) Due to the communication constraint which is described by a directed communication graph, we cannot use the full information of the system for feedback control, and we have to develop distributed control laws to deal with the tracking problem. The distributed tracking control design needs to consider the interactions among agents, coupling terms in dynamics, and the capability on information collection of each agent and so on, which makes the controller design and performance analysis of the closed-loop systems much more difficult.

The rest of this paper is organized as follows. Section 2 is on notation. Section 3 is for problem formulation. Section 4 presents distributed controllers design and analyzes the performance properties. Section 5 applies the theoretical results to mechanical multi-agent systems. Section 6 includes some concluding remarks.

2. Notation

The following notation will be used throughout this paper. For a given vector or matrix X, X^T denotes its transpose. $|X|$ is the Euclidean norm of a vector X. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted digraph of order n with the set of nodes $\mathcal{V} = \{1, 2, \ldots, n\}$, set of arcs $\mathcal{E} \subset \mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $A = (a_{ij})_{n\times n}$ with nonnegative elements. $(j, i) \in \mathcal{E}$ means that agent j can directly send information to agent i. In this case, j is called the parent of i, and i is called the child of j. The set of neighbors of vertex i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, i \neq j\}$. $a_{ij} > 0$ if node j is a neighbor of node i and $a_{ij} = 0$ otherwise. In this paper, we assume that there is no self-loop, i.e. $a_{ii} = 0$. Node i is called an isolated node, if it has neither parent nor child. Node i is called a source if it has no parents but children. Denote the sets of all sources and isolated nodes in $\mathcal{V}$ by $\mathcal{V}_s = \{j \in \mathcal{V}|N_j = \emptyset,\ \emptyset$ is the empty set$\}$. To avoid the trivial cases, $\mathcal{V} - \mathcal{V}_s \neq \emptyset$ is always assumed in this paper. A sequence $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ of edges is called a directed path from node i_1 to node i_k. A directed tree is a digraph, where every node except the root has exactly one parent and the root is a source. A spanning tree of $\mathcal{G}$ is a directed tree whose node set is $\mathcal{V}$ and whose edge set is a subset of $\mathcal{E}$. The diagonal matrix $D = \mathrm{diag}(\kappa_1, \kappa_2, \ldots, \kappa_n)$ is the degree matrix, whose diagonal elements $\kappa_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. The Laplacian of a weighted digraph $\mathcal{G}$ is defined as $L = D - A$.

We consider a system consisting of n agents and a leader (labeled by 0) which is depicted by a graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \{0, 1, 2, \ldots, n\}$, set of arcs $\bar{\mathcal{E}} \subset \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. If $(0, i) \in \bar{\mathcal{E}}$, then $0 \in \mathcal{N}_i$. A diagonal matrix $B = \mathrm{diag}(b_1, b_2, \ldots, b_n)$ is the leader adjacency matrix associated with $\bar{\mathcal{G}}$, where $b_i > 0$ if node 0 is a neighbor of node i; and $b_i = 0$, otherwise.

Lemma 1 ([18]). *Let x, y be real variables. For any positive real numbers m and n, and positive real numbers a and b, the following inequality holds:*

$$ax^m y^n \le b|x|^{m+n} + \frac{n}{m+n}\left(\frac{m+n}{m}\right)^{-m/n} a^{(m+n)/n} b^{-m/n} |y|^{m+n}.$$

3. Problem formulation

Consider the following high-order nonlinear MASs (the followers) described by:

$$\begin{aligned}
\dot{x}_{ij} &= x_{i,j+1}^{p_{ij}} + f_{ij}(\bar{x}_{ij}), \quad j = 1, \ldots, n_i - 1,\\
\dot{x}_{i,n_i} &= u_i^{p_{i,n_i}} + f_{i,n_i}(\bar{x}_{i,n_i}),\\
y_i &= x_{i1}, \quad i = 1, \ldots, N,
\end{aligned} \tag{1}$$

where $\bar{x}_{i,j} = (x_{i1}, \ldots, x_{i,j})^T \in R^j$, $u_i \in R$, $y_i \in R$ are the state, input, output of system, respectively; $p_{ij} \in R^{\ge 1}_{odd} = \{q \in R : q \ge 1$ and q is a ratio of odd integers$\}$. The functions f_{ij} are smooth. The leader's output is denoted as $y_0(t)$.

The following assumptions are made on the system (1).

Assumption 1. $p_{11} = p_{21} = \cdots = p_{N1} = 1$.

Assumption 2. $\bar{\mathcal{G}}$ contains a directed spanning tree with the leader as the root.

Assumption 3. The leader's output $y_0(t) \in R$ and $\dot{y}_0(t)$ are bounded and available for the ith followers satisfying $0 \in N_i$, $i = 1, \ldots, N$.

Remark 1. In the following, we give an example to discuss the relationship between the tracking control algorithm and the Jacobian linearization method. Without loss of generality, we only consider the case of single-agent system since MASs is in a similar way.

Consider a three-order nonlinear system

$$\begin{aligned}
\dot{x}_1 &= x_2 + x_1^2,\\
\dot{x}_2 &= x_3^3,\\
\dot{x}_3 &= u^5,\\
y &= x_1.
\end{aligned} \tag{2}$$

We aim to find the possible control scheme to make y track the given signal $y_r(t) \equiv 1$ asymptotically while keeping all the states bounded.

Let $e = y - y_r(t) = x_1 - 1$, then (2) can be written as

$$\begin{aligned}
\dot{e} &= x_2 + (e + 1)^2,\\
\dot{x}_2 &= x_3^3,\\
\dot{x}_3 &= u^5.
\end{aligned} \tag{3}$$

The linearized system of (3) at (0, 0) is

$$\dot{x} = Ax + Bu,$$

where

$$A = \begin{bmatrix} 2 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix},$$

which is uncontrollable and the uncontrollable mode has a positive eigenvalue 2. By Theorem 1 in [19], system (3) cannot be asymptotically stabilized by any continuously differentiable control.

From the above discussion, we can find that it is challenging to achieve the tracking control objective of single-agent nonlinear systems with unstable linearization parts. Besides this, for multi-agent systems, the distributed tracking control design also has to simultaneously consider the interactions among agents, coupling terms in dynamics, and the capability on information collection of each agent and so on, which makes the controller design and performance analysis of the closed-loop systems much more difficult. Being aware of that it is impossible to deal with the