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Transitivity in simultaneous stabilization for a family of time-varying systems[☆]

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ABSTRACT

This paper develops necessary and sufficient conditions for the so-called transitivity and strong transitivity (precise definitions later) for a family of discrete time-varying linear systems, which when satisfied lead to characterizations for simultaneous stabilizability and simultaneously stabilizing feedback controllers. Some criteria for the strong transitivity are established in terms of single coprime factorizations of only one plant and one controller, the resulting characterization involves coprime factorizations of fewer systems compared to previous work.

1. Introduction

The simultaneous stabilization problem concerns the design of a common controller that stabilizes a finite set of plants. The first explicit statement of the problem of finding necessary and sufficient conditions for the existence of a simultaneously stabilizing controller was made by Saeks and Murray [1]. Necessary and sufficient conditions for the existence of a simultaneously stabilizing controller for two plants were developed by Vidyasagar and Viswanadham [2]. However, the problem of finding necessary and sufficient conditions for the existence of simultaneously stabilizing controllers for three or more plants remained unsolved. In [3], Blondel showed that, unlike for two plants, there does not exist necessary and sufficient conditions for simultaneous stabilization when the number of plants is strictly greater than two, only sufficient or necessary conditions may be found.

Simultaneous stabilization problem has been studied in various frameworks. In this paper, we consider the simultaneous stabilization of a finite class of time-varying linear system within the framework of nest algebra. As the development of H_∞ control theory, a great deal of insight can be gained by considering its time-varying analogue on an appropriate complex Hilbert space of input–output signals. One approach involves an input–output perspective, namely, a causal time-varying linear system is considered as a lower-triangular operator (possibly unbounded) defined on a certain separable Hilbert space, where the algebra of stable, causal, time-varying linear systems is represented by the nest algebra. This approach is a generalization of H_∞ control in the sense that 2-norm is used to quantify the size of signals, the mathematical framework will be operator theoretic as described in [4]. This interpretation of dynamical systems has generated significant research interest over the last decade, as witnessed by recent contributions such as [5–10]. For the time-varying linear systems the stabilization problem was first (as far as we know) formulated in [11], the class of all causal time-varying linear systems which are stabilizable has been shown to be those which allow doubly coprime factorizations and there is a Youla type parametrization of all stabilizers which is conceptually similar to the classical result for linear time-invariant systems. Readily checked necessary or sufficient conditions for the existence of simultaneous stabilizing controllers for a family of time-varying systems are known, see, e.g. [4,12]. The common point of reference is the well-known Youla characterization. The construction of such simultaneous stabilizing controllers involves the satisfaction of constraints on coprime factorizations of every given plant. However, few necessary and sufficient conditions appeared in this direction without any restrictions to the plants. The simultaneous stabilization problem for three systems is recognized as one of the open problems in linear system theory, many authors try to solve this problem under some restrictions. In [13], Yu brings out a new point of view to discuss the simultaneous stabilization problem for three plants, that

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is, "transitivity in simultaneous stabilization". The transitivity restricts the search for the existence of a simultaneously stabilizing controller to the plants satisfying a so-called transmission condition. The *transmission condition* for three given plants L_0, L_1, L_2 is "L_0, L_1 can be simultaneously stabilized by some system C_0, and L_1, L_2 can be simultaneously stabilized by some system C_1". The family of such plants L_0, L_1 and L_2 has a *transitivity* property in simultaneous stabilization if L_0 and L_2 can be simultaneously stabilized. While, L_0, L_1 and L_2 satisfy *strong transitivity* if L_0, L_1 and L_2 are simultaneously stabilizable under the transmission condition.

It is noted that the traditional necessary and sufficient conditions for simultaneous stabilization are all characterized in the form of coprime factorizations of every given plant. However, it is still open how to compute the coprime factorization of a general time-varying linear system. Fortunately, the transmission condition can be seen as a weaker replacement of coprime factorization condition. In [14], the authors give a simultaneous stabilization criterion for three time-varying plants under the transmission condition, the criterion is in terms of coprime factorizations of only two plants but not all. The transitivity idea is in fact an interesting contribution to the important area of simultaneous stabilization problem. In the context, it is natural to ask whether the transitivity idea can be extended to a family of $n(n > 3)$ plants, and whether among these simultaneous stabilizability criteria there exists one in terms of coprime factorizations of fewer plants.

The purpose of this paper is to deal with the transitivity and strong transitivity in simultaneous stabilization for a finite family of plants based on coprime factorization theory. The task is to conveniently design the simultaneous stabilizability criteria in terms of coprime factorizations of fewer plants, thereby generalizing Youla parametrization theory for simultaneously stabilizing controllers [11,14]. First, we derive a necessary and sufficient condition to the transitivity in simultaneous stabilization of a family of time-varying linear plants $L_0, L_1, \ldots, L_n$. Furthermore, a new design method is derived for all simultaneously stabilizing controllers under the restrictive transmission condition, which only requires a left coprime factorization of one given plant. Lastly, we point out that the results obtained for the strong transitivity apply to simultaneous stabilization as well without resorting to coprime factorizations of $L_0, L_2, \ldots, L_n$. In particular, we rebuild some criteria for transitivity and strong transitivity of three plants, which extend results in [14] by only requiring a coprime factorization of one plant rather than two plants.

This paper evolves along the following line. The mathematical background on the time-varying linear systems and nest algebra are given in Section 2. In Section 3, a new necessary and sufficient condition to the transitivity in simultaneous stabilization of n $(n \geq 3)$ plants is derived. Section 4 considers the simultaneous stabilization of a finite family of plants under the transmission condition. A numerical example is presented in Section 5. Section 6 concludes the paper.

2. Preliminaries

Let $\mathbb{C}$ denote the set of complex numbers and $\mathbb{C}^d$ the Cartesian product of d copies of $\mathbb{C}$, here d is a fixed positive integer. The signal space considered in this paper is

$$\mathcal{H} = \left\{ (x_0, x_1, x_2, \ldots) : x_i \in \mathbb{C}^d, \ \sum_{i=0}^{\infty} \|x_i\|_d^2 < \infty \right\},$$

where $\|\cdot\|_d$ denotes the standard Euclidean norm on $\mathbb{C}^d$. $\mathcal{H}$ is the complex Hilbert space with the scalar product and inner product in the following form:

$$\alpha x = (\alpha x_0, \alpha x_1, \alpha x_2, \ldots),$$

$$\langle x, y \rangle = \sum_{i=0}^{+\infty} \langle x_i, y_i \rangle_{\mathbb{C}^d}, \quad \alpha \in \mathbb{C}, x, y \in \mathbb{C}^d.$$

$\mathcal{H}_e = \{(x_0, x_1, \ldots) : x_i \in \mathbb{C}^d\}$ will denote the extended space of $\mathcal{H}$.

For each $n \geq 0$, we denote by P_n the standard truncated projection on $\mathcal{H}$ or $\mathcal{H}_e$ as

$$P_n(x_0, x_1, \ldots, x_n, \ldots) = (x_0, x_1, \ldots, x_n, 0, \ldots)$$

with $P_{-1} = 0$ and $P_\infty = I$.

$\{P_n : -1 \leq n \leq +\infty\}$ is used to define the physical definition of causality. A linear transformation L on $\mathcal{H}_e$ is *causal* if $P_n L P_n = P_n L$ for each n. L is a (time-varying) linear system if it is a causal linear transformation on $\mathcal{H}_e$ and continuous with respect to the standard seminorm topology (Chapter 5, [4]). Indeed, any linear system is an infinite-dimensional lower triangular matrix with respect to the standard basis of $\mathcal{H}_e$. A linear system is *stable* if its restriction to $\mathcal{H}$ is a bounded operator.

Let $\mathcal{L}$ be the algebra of linear systems with respect to the standard addition and multiplication. The set of stable ones, denoted by $\mathcal{S}$, referred to in the operator theory literature as a nest algebra determined by the nest $\{I - P_n : -1 \leq n \leq \infty\}$ (Chapter 3, [4]). An important subalgebra of $\mathcal{L}$ is the algebra $\mathcal{T}$ of time-invariant linear systems. This algebra consists of the lower matrices that are Toeplitz (although not necessarily bounded operators). It is easily seen that $\mathcal{T} \cap \mathcal{S} = H_\infty$.

The set of invertible elements in $\mathcal{S}$ is denoted by $\mathrm{U}(\mathcal{S})$.

Consider the feedback configuration contributed by the plant $L \in \mathcal{L}$ and the controller $C \in \mathcal{L}$, where the closed-loop equation is

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} I & C \\ -L & I \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}.$$

The closed-loop system $\{L, C\}$ is *well posed* if the internal input $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$ can be expressed as a causal function of the external input $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. This is equivalent to requiring that $\begin{bmatrix} I & C \\ -L & I \end{bmatrix}$ being invertible. The inverse is given by the transfer matrix

$$H(L, C) = \begin{bmatrix} (I + CL)^{-1} & -C(I + LC)^{-1} \\ L(I + CL)^{-1} & (I + LC)^{-1} \end{bmatrix}.$$

Definition 2.1. (1) The closed-loop system $\{L, C\}$ is *stable* if each entry of $H(L, C)$ belongs to $\mathcal{S}$.

(2) A plant L is *stabilizable* if there exists a linear system $C \in \mathcal{L}$ such that $\{L, C\}$ is stable. In this case, C is said to be a *stabilizing controller* for L.

(3) A family of plants $L_0, L_1, \ldots, L_n$ are *simultaneously stabilizable* if there exists a linear system $C \in \mathcal{L}$ such that $\{L_i, C\}$ is stable for all $i = 0, 1, \ldots, n$.

In order to characterize the stabilizability and simultaneous stabilizability, the notions of right and left coprime factorizations for the time-varying linear systems are needed.

Definition 2.2. Let $M, N, \hat{M}, \hat{N} \in \mathcal{S}$ and $L \in \mathcal{L}$.

1. NM^{-1} is a *right coprime factorization* of L if $L = NM^{-1}$ with M invertible in $\mathcal{L}$, and there exist $X, Y \in \mathcal{S}$ such that $\begin{bmatrix} Y & X \end{bmatrix} \begin{bmatrix} M \\ N \end{bmatrix} = I$.
2. $\hat{M}^{-1}\hat{N}$ is a *left coprime factorization* of L if $L = \hat{M}^{-1}\hat{N}$ with $\hat{M}$ invertible in $\mathcal{L}$, and there exist $\hat{X}, \hat{Y} \in \mathcal{S}$ such that $\begin{bmatrix} -\hat{N} & \hat{M} \end{bmatrix} \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix} = I$.

Lemma 2.1 ([4, Theorem 6.3.5])**.** *Assume that NM^{-1} is a right coprime factorization of L and $\hat{M}^{-1}\hat{N}$ is a left coprime factorization of L. Then any right coprime factorization of L is of the form $(NZ)(MZ)^{-1}$ with $Z \in \mathrm{U}(\mathcal{S})$; and any left coprime factorization of that is of the form $(Z\hat{M})^{-1}(Z\hat{N})$ with $Z \in \mathrm{U}(\mathcal{S})$.*