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Convex conditions for robust stabilization of uncertain switched systems with guaranteed minimum and mode-dependent dwell-time

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ABSTRACT

Alternative conditions for establishing dwell-time stability properties of linear switched systems are considered. Unlike the hybrid conditions derived in Geromel and Colaneri (2006), the considered ones are affine in the system matrices, allowing then for the consideration of uncertain switched systems with time-varying uncertainties. The low number of decision variables moreover permits to easily derive convex stabilization conditions using a specific class of state-feedback control laws. The resulting conditions are enforced using sum of squares programming which are shown to be less complex numerically that approaches based on piecewise linear functions or looped-functionals previously considered in the literature. The sums of squares conditions are also proven to (1) approximate arbitrarily well the conditions of Geromel and Colaneri (2006); and (2) be invariant with respect to time-scaling, emphasizing that the complexity of the approach does not depend on the size of the dwell-time. Several comparative examples illustrate the efficiency of the approach.

1. Introduction

Switched systems [1–9] are very flexible modeling tools appearing in several fields such as switching control laws [4,10], networked control systems [11], electrical devices/circuits [12,13], and congestion modeling and control in networks [14–16]. When switching between a family of asymptotically stable subsystems holds in a way that is independent of the state of system, stability under minimum and average dwell-times have been shown to be relevant concepts of stability [17,1] for which certain criteria have been proposed. Hybrid conditions, consisting of joint continuous-time and discrete-time conditions, for characterizing minimum dwell-time have been recently proposed in [5] where it is shown that the use of quadratic Lyapunov functions may lead to better results than previous ones. Even more importantly, homogeneous Lyapunov functions have been proved to be able to formulate non-conservative conditions for minimum dwell-time analysis [18,19]. However, extending these important results to uncertain systems, time-varying systems and control design is quite difficult due to the presence of exponential terms that are not applicable to time-varying systems and would create strongly nonconvex terms in the design conditions.

Looped-functionals [20–23], on the other hand, are a particular class of indefinite functionals (i.e. not required to be positive definite) satisfying a looping-condition—a particular boundary algebraic condition. They have been shown to yield stability conditions that are less conservative than those obtained using positive definite Lyapunov functionals; see e.g. [24,21,20]. They have also been shown to provide an alternative framework for dwell-time analysis of switched systems which remains compatible with uncertain switched systems, time-varying subsystems and, potentially, nonlinear switched systems. Tractable conditions for robust stability analysis under mode-dependent dwell-time, a stability concept permitting the instability of the subsystems [23], can be obtained as well using such a framework. However, the structure of the conditions and the large number of decision variables make the derivation of computationally attractive synthesis conditions a hardly possible task.

The approach proposed in this paper is based on clock-dependent Lyapunov functions, a class of Lyapunov functions explicitly depending on the time elapsed since the last discrete-time event (i.e. a clock); see e.g. [9,25–28]. They have been applied to switched systems [9,26,28], sampled-data systems [9,25,27] and impulsive systems [9,27]. The advantages of clock-dependent Lyapunov functions lie in the absence of any exponential term, facilitating the derivation of tractable conditions for establishing the stability of uncertain hybrid systems. The advantages over the use of looped-functionals are a lower computational complexity and

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the possibility of deriving convex conditions for the control of hybrid systems via state-feedback.

The contribution of the paper is manifold. First, alternative minimum dwell-time stability conditions, rigorously shown to be equivalent to those obtained in [5], are provided. The advantage of the proposed conditions lies in their affine dependence in the system matrices, permitting then their extension to uncertain systems with time-varying subsystems, as opposed to the conditions of [5] that are only applicable to LTI subsystems. The price to pay, however, is the characterization of stability with minimum dwell-time using infinite-dimensional convex semidefinite programs, which may be hard to solve when the considered system is of large dimension. A piecewise linear approximation of these conditions have been proposed in [26] and results in a finite-number of linear matrix inequalities. However, as it will be emphasized later, the discretization order often needs to be large in order to obtain accurate results. In contrast, the sum of squares approach [29,30] considered in this paper yields more accurate results while being faster and computationally less expensive than the piecewise-linear approach [26] and the approach based on looped-functionals [23]. It is also proven that, for the class dwell-time conditions we consider, the sum of squares relaxation is asymptotically exact, meaning that by choosing a sufficiently large polynomial order, the conditions based on sum of squares approximate arbitrarily well the conditions of [5]. A result proving the invariance of the sum of squares conditions is also proved and shows that the polynomial order is independent of the minimum dwell-time value and only depends on the matrices of the switched system.

Outline: The structure of the paper is as follows: in Section 2 preliminary definitions and results are given. Section 3 is devoted to minimum dwell-time stability analysis whereas Section 4 addresses stability under mode-dependent dwell-time. Results on the stabilization under minimum and mode-dependent dwell-time are derived in Section 5.

Notations: The sets of symmetric and positive definite matrices of dimension n are denoted by $\mathbb{S}^n$ and $\mathbb{S}^n_{\succ 0}$ respectively. Given two symmetric real matrices A and B, the inequalities $A \succ (\succeq) B$ mean that $A - B$ is positive (semi)definite. For any square matrix M, we define $\mathrm{Sym}[M] = M + M^\intercal$.

2. Preliminaries

2.1. System definition

From now on, the following class of linear switched system

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) \\ x(t_0) &= x_0 \end{aligned} \tag{1}$$

are considered where $x, x_0 \in \mathbb{R}^n$ are the state of the system and the initial condition, respectively. The switching signal σ is defined as a left-continuous piecewise constant function $\sigma : [0, \infty) \to \{1, \ldots, N\}$. At some point, the matrices A_i of the subsystems will be uncertain and/or time-varying, this will be explicitly stated when this is the case. We also assume that the sequence of switching instants $\{t_1, t_2, \ldots\}$ is increasing and does not admit any accumulation point. Consequently, any Zeno motion is excluded.

2.2. Stability with periodic switching times

We start with a stability result under periodic switching that allows us to state the main ideas in a simple context. By periodic switching, it is meant here that switching times are periodic, i.e. $t_{k+1} = t_k + \bar{T}$, for some $\bar{T} > 0$. Note, however, that the sequence of subsystems is not necessarily periodic and, thus, periodic systems theory does not apply here. The following result will be shown to be directly involved in the derivation of the results on minimum dwell-time stability in the next section.

Theorem 1 (*Stability with Periodic Switching Times*). *The following statements are equivalent:*

(a) *The quadratic form $V(x(t), \sigma(t)) = x(t)^\intercal P_{\sigma(t)} x(t)$, $P_i \in \mathbb{S}^n_{\succ 0}$, $i = 1, \ldots, N$, is a discrete-time Lyapunov function for the switched system (1) with $\bar{T}$-periodic switching times in the sense that the inequality*

$$V(x(t_{k+1}), \sigma(t_{k+1})) - V(x(t_k), \sigma(t_k)) \le -\mu \|x(t_k)\|_2^2 \tag{2}$$

holds for some $\mu > 0$, all $x(t_k) \in \mathbb{R}^n$ and all $k \in \mathbb{N}$.

(b) *There exist matrices $P_i \in \mathbb{S}^n_{\succ 0}$, $i = 1, \ldots, N$ such that the LMIs*

$$e^{A_i^\intercal \bar{T}} P_i e^{A_i \bar{T}} - P_j \prec 0 \tag{3}$$

hold for all $i, j = 1, \ldots, N$, $i \neq j$.

(c) *There exist differentiable matrix functions $R_i : [0, \bar{T}] \mapsto \mathbb{R}^n$, $R_i(0) \in \mathbb{S}^n_{\succ 0}$, $i = 1, \ldots, N$, and a scalar $\varepsilon > 0$ such that the LMIs*

$$A_i^\intercal R_i(\tau) + R_i(\tau) A_i - \dot{R}_i(\tau) \preceq 0 \tag{4}$$

and

$$R_i(\bar{T}) - R_j(0) + \varepsilon I \preceq 0 \tag{5}$$

hold for all $\tau \in [0, \bar{T}]$ and all $i, j = 1, \ldots, N$, $i \neq j$.

(d) *There exist differentiable matrix functions $S_i : [0, \bar{T}] \mapsto \mathbb{S}^n$, $S_i(\bar{T}) \in \mathbb{S}^n_{\succ 0}$, $i = 1, \ldots, N$, and a scalar $\varepsilon > 0$ such that the LMIs*

$$A_i^\intercal S_i(\tau) + S_i(\tau) A_i + \dot{S}_i(\tau) \preceq 0 \tag{6}$$

and

$$S_i(0) - S_j(\bar{T}) + \varepsilon I \preceq 0 \tag{7}$$

hold for all $\tau \in [0, \bar{T}]$ and all $i, j = 1, \ldots, N$, $i \neq j$.

Proof. *Proof of* (a) $\Leftrightarrow$ (b): Assume $\sigma(t_k) = j$ and $\sigma(t_k + \tau) = i$, $\tau \in (0, \bar{T}]$. Then, we have

$$\begin{aligned} &V(x(t_{k+1}), \sigma(t_{k+1})) - V(x(t_k), \sigma(t_k)) \\ &\quad = x(t_k)^\intercal \left[e^{A_i^\intercal \bar{T}} P_i e^{A_i \bar{T}} - P_j \right] x(t_k) \end{aligned} \tag{8}$$

and there exists $\mu > 0$ such that (2) holds if and only if (3) holds. The proof is complete.

Proof of (c) $\Rightarrow$ (b): Assume (c) holds. Solving (4) for $R_i(\tau)$ yields [31]

$$R_i(\tau) \succeq e^{A_i^\intercal \tau} R_i(0) e^{A_i \tau} \tag{9}$$

and thus

$$e^{A_i^\intercal \bar{T}} R_i(0) e^{A_i \bar{T}} - R_i(\bar{T}) \preceq 0. \tag{10}$$

From (5), we have that $R_i(\bar{T}) \preceq R_j(0) - \varepsilon I$ and therefore, combining this with (10), we obtain

$$e^{A_i^\intercal \bar{T}} R_i(0) e^{A_i \bar{T}} - R_j(0) + \varepsilon I \preceq 0 \tag{11}$$

which implies in turn that (3) holds with $P_i = R_i(0)$.

Proof of (d) $\Rightarrow$ (b): The proof follows the same lines as the one above and is omitted. Note that, in this case, (3) holds with $P_i = S_i(\bar{T})$.

Proof of (b) $\Rightarrow$ (c): The idea of the proof is to show that, when there exists matrices $P_i \in \mathbb{S}^n_{\succ 0}$, $i = 1, \ldots, N$, such that the LMI (3) holds, then the LMIs (4)–(5) hold with the matrix-valued functions $R_i(\tau) = R_i^*(\tau)$ with $R_i^*(\tau) = e^{A_i^\intercal \tau} P_i e^{A_i \tau}$.

Computing then the derivative of $R_i^*(\tau)$ with respect to τ and noting that $A_i^\intercal e^{A_i^\intercal \tau} P_i e^{A_i \tau} = A_i^\intercal R_i^*(\tau)$, we obtain that $-\dot{R}^*(\tau) + A_i^\intercal R_i^*(\tau) + R_i^*(\tau) A_i = 0$ for all $\tau \in [0, \bar{T}]$; hence (4) holds. Noting now that the condition (3) is equivalent to saying that there exists