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Iterative learning control of inhomogeneous distributed parameter systems—frequency domain design and analysis[☆]

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ABSTRACT

This paper aims to construct a design and analysis framework for iterative learning control of linear inhomogeneous distributed parameter systems (LIDPSs), which may be hyperbolic, parabolic, or elliptic, and include many important physical processes such as diffusion, vibration, heat conduction and wave propagation as special cases. Owing to the system model characteristics, LIDPSs are first reformulated into a matrix form in the Laplace transform domain. Then, through the determination of a fundamental matrix, the transfer function of LIDPS is precisely evaluated in a closed form. The derived transfer function provides the direct input–output relationship of the LIDPS, and thus facilitates the consequent ILC design and convergence analysis in the frequency domain. The proposed control design scheme is able to deal with parametric and non-parametric uncertainties and make full use of the process repetition, while avoid any simplification or discretization for the 3D dynamics of LIDPS in the time, space, and iteration domains. In the end, two illustrative processes are addressed to demonstrate the efficacy of the proposed iterative learning control scheme.

1. Introduction

Iterative learning control (ILC) is a mature learning control strategy by fully utilizing the past control experience to improve the current tracking performance. It is developed for control tasks that repeat in a fixed time interval, and requires only the system gradient bounds instead of accurate system model. ILC is initially proposed in 1984 [1], and now has been well established in terms of both the underlying theory and experimental applications [2–5]. The main research trends in this field include ILC for non-repetitive tasks or plants, non-smooth nonlinearities, as well as infinite-dimensional systems, etc. [6].

Currently, the vast majority of the work reported on ILC considers finite-dimensional systems but there has been some work reported on ILC of distributed parameter systems (DPSs) governed by partial differential equations (PDEs). In [7], an iterative learning approach is applied for the constrained digital regulation of a class of linear hyperbolic PDE systems, where the plant model is first reduced to ordinary differential equation (ODE) systems and then approximated by the discrete-time equivalence. In [8], ILC scheme is presented for more general spatio-temporal dynamics using *n*D discrete linear system models. Without any discretization of system, [9] considers the design of P-type and D-Type ILC laws for a class of infinite-dimensional linear systems using semigroup theory. It is worthy of noticing that the aforementioned three works all adopt distributed control structure, namely, the number of control actuators is more than one and they are uniformly distributed along the spatial domain. Further, to address the application of ILC for some specific DPSs, [10] considers ILC of flow rate in a center pivot irrigator used in dry-land farming, which can be modeled as a spatial–temporal diffusion process in three spatial dimensions coupled with flow in one dimension. In [11], based on Lyapunov theory, differential-difference type ILC is augmented with proportional controller to attenuate the unknown periodic speed

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variation for a stretched string system on a transporter. In [12], the similar ILC scheme is combined with proportional-derivative controller to compensate for the unknown periodic motion on the right end for a class of axially moving material systems. In [11,12], ILC is mainly designed for the stability maintenance of mechanical processes. Recently, under the framework of ILC, velocity boundary control of a quasi-linear PDE process is considered in [13], where the convergence of output regulation is guaranteed in the steady-state stage. Investigating all the available results in this field, ILC for infinite-dimensional processes demonstrates clear differences to ILC for finite-dimensional processes in design and analysis, e.g., the infinite-dimensional characteristic of system, the interweave of 3D dynamics in the time, space, and iteration domains, and the absence of universal analysis tools in convergence analysis [14].

The study of the paper is motivated by the following facts. First, up to the present, all the references that address ILC of linear and nonlinear PDEs are focusing on some *specific* processes, where the controller design highly depends on the properties of system model. For instance, in [14], the boundary control of a class of inhomogeneous heat equation is considered, and the ILC convergence analysis is highly depending on the explicit solution of the nonlinear system in the time domain. However, it is not clear how to extend the main idea in [14] to other types of DPS processes. Second, many industrial and engineering processes can be described by linear or linearized PDE models, although nonlinear PDEs would have been of interest from a practical viewpoint [15]. Meanwhile, the involved system parameters or even inhomogeneous source terms may change with operating conditions. Third, parametric or non-parametric uncertainties can be dealt with by ILC easily under repetitive control environment, owing to the model-free nature in the design process of learning controller [16]. In association with the above observations, this paper aims at ILC design and analysis for general LIDPSs that may be hyperbolic, parabolic, or elliptic, and include many important physical processes such as diffusion, vibration, heat conduction and wave propagation as special cases. In order to overcome the difficulties that are associated with ILC of LIDPSs, the system equations are first reformulated into a matrix form in the Laplace transform domain. Through determination of a fundamental matrix, the system transfer function is then precisely evaluated in a closed form. The transfer function of a LIDPS contains all information required to predict the system spectrum, the system response under any initial and external disturbances, and the stability of the system response. Meanwhile, the derived transfer function clearly demonstrates the input–output relationship of system, and thus facilitates the consequent ILC design and convergence analysis in the frequency domain. As a result, one can iteratively tune the boundary input condition such that the output at the concerned position can track the desired reference pointwisely. Meanwhile, owing to the fact that ILC is a feedforward control, the proposed scheme not only makes anticipatory compensation possible to overcome the time delay in boundary output tracking, but also eliminates the gain margin limitation encountered in feedback control.

The main contributions of the paper are summarized as follows.

(i) A uniform design and analysis framework is presented for ILC of LIDPSs in the frequency domain. Nevertheless, [7–14] consider the ILC of LIDPSs or DPSs all in the time domain.
(ii) Instead of simplifying the infinite-dimensional PDEs to finite-dimensional ODEs and/or replacing them by the discrete-time equivalences as in [7,8,10], the model approximation problem is avoided in controller design. Thus the often physically motivated model is advantageously maintained throughout the entire control design process. In doing so, non-physically motivated parameters, like discretization parameters are avoided [17].
(iii) Different from [7–9] that use a distributed control structure, we consider LIDPSs with point (boundary) control, namely, both the input actuator and the output sensor are unique. Such scenario is more practical and implementable in certain applications [18–20].
(iv) Instead of considering the stability or set-point problem as in [7,11–13], we consider more general output tracking problem.

The paper is organized as follows. In Section 2, problem formulation is first given. In Section 3, we present the details for calculating the input–output transfer functions for the considered LIDPSs. In consequence, based on the derivation in Section 3, we focus on ILC design and convergence analysis in Section 4. Then, Section 5 addresses the robustness problem of the ILC scheme. At last, an illustrative example is presented in Section 6.

2. Problem formulation

Consider the one-dimensional, nth-order, linear inhomogeneous PDE under a repeatable process environment

$$\left(A\frac{\partial^2}{\partial t^2}+B\frac{\partial}{\partial t}+C\right)w^i(x,t)=f(x,t)+g^i(x,t), \tag{1}$$

where the time $t \in (0, T]$ for any fixed $T > 0$, the spatial coordinate $x \in (0, 1)$, $i \in \mathcal{Z} \triangleq \{0, 1, 2, \ldots\}$ is the trial or iteration number, and $w^i(x, t)$ represents the system state in the ith iteration that may be interpreted as temperature in a heat transfer process or pollutant concentration in a wastewater treatment process. Meanwhile, the *unknown* nonlinear functions $f(x, t)$ and $g^i(x, t)$ denote the iteration-independent and iteration-dependent external disturbances, respectively. Moreover, A, B and C are spatial differential operators of the form $A=\sum_{k=0}^n a_k\frac{\partial^k}{\partial x^k}$, $B=\sum_{k=0}^n b_k\frac{\partial^k}{\partial x^k}$, $C=\sum_{k=0}^n c_k\frac{\partial^k}{\partial x^k}$ with a_k, b_k and c_k being constants and satisfying $|a_k|^2+|b_k|^2+|c_k|^2\neq 0$ as $k = n$. For the system (1), the boundary conditions are set as, for $t \in [0, T]$, $i \in \mathcal{Z}$, $1 \le j \le n$,

$$\begin{cases} M_jw^i(0,t)+N_jw^i(1,t)=\gamma_j(t), & j\neq j_0,\\ M_jw^i(0,t)+N_jw^i(1,t)=u^i(t), & j=j_0,\end{cases} \tag{2}$$

where $1 \le j_0 \le n$ is a fixed integer, and M_j, N_j are temporal–spatial, linear differential operators of proper order. The functions $\gamma_j(t), j \neq j_0$ are unknown but iteration-invariant, while u^i is the tunable system control input. Meanwhile, the initial conditions of system (1) for all $x \in (0, 1)$ are specified as

$$w(x,t)|_{t=0}=v_0(x),\qquad \frac{\partial}{\partial t}w(x,t)|_{t=0}=v_1(x), \tag{3}$$

where $v_0(x)$ and $v_1(x)$ are given continuous functions. To validate our consequent ILC design and analysis, we assume that the boundary value problem (1)–(3) is well posed, and always has one and only one solution.

It is worth highlighting that the system (1) may be hyperbolic, parabolic, or elliptic, and describes many important physical processes such as diffusion, heat transfer, vibration, wave propagation, etc. For instance, in describing vibration of a continuum, (1) is of hyperbolic type, the operator $A\partial^2/\partial t^2$ is associated with the inertia properties of the continuum, the operator $B\partial/\partial t$ evolves from damping, Coriolis acceleration, and mass transport, and the operator C is relevant to stiffness, centrifugal forces, and circulatory effects [21].

Consider a point control problem for the system (1), namely, iteratively tuning the boundary input condition $u^i(t)$ such that the output $y^i(t) = w^i(x^*, t)$, $t \in [0, T]$ can track the given reference trajectory $y^d(t)$, $t \in [0, T]$, where $0 \le x^* \le 1$ is the spatial position of the measurement output. Clearly, when $x^* = 0$ or $x^* = 1$,