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Distributed control for alpha-heterogeneous dynamically coupled systems

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ABSTRACT

This paper concerns the problem of distributed controller synthesis for a class of heterogeneous distributed systems composed of α (2 or more) different kinds of subsystems, interacting with one another according to a certain given graph topology. We will show that by employing Linear Matrix Inequalities (LMIs) tools, namely the full-block S-procedure, we can derive a control synthesis method based on $\mathcal{L}_2$ gain performance. This synthesis method guarantees stability and performance of a whole set of possible interconnection graphs, and its computational complexity does not depend on the number of subsystems involved but only on the number of different kinds of subsystems. The effectiveness of the new method is verified on a test case.

1. Introduction

The system and control community is devoting significant efforts on the development of distributed control methods for large scale systems, as it can be seen from the large number of works published in the field in the last 40 years [1–7]. By "distributed control", opposed to "centralized control", we mean a control action that is computed locally according to the physical spatial extension of the system, which is seen as an interconnection of simpler subsystems. The goal is replacing the high-order centralized controller with many simple (low-order) elementary controllers which only have access to a limited set of measures, for example only to those of the subsystems to which they are physically attached and their nearest neighbors.

This paper concerns the control of heterogeneous systems (see for example [5,8,9] and references therein), made by the interconnection of N subsystems (or agents), according to an interconnection structure described by a graph. We make the additional hypothesis of a certain regularity, namely we restrict to heterogeneous systems that are only made of a limited number α of different subsystem types, as shown in Fig. 1. For such systems, which we will call "α-heterogeneous", we will consider the interconnection as an uncertainty (as done for example in [10]), and we will use a robust control tool (the full-block S-procedure [11]) to deal with it in the form of LMIs (Linear Matrix Inequalities). The contribution of this article is the fact that we show that such LMIs can be reduced to a set whose size does not depend on the number of subsystems N, but only on α. This means that we obtain an analysis and synthesis method whose computational complexity is not depending on the number of subsystems, and which can virtually be applied even for $N \to \infty$, as long as the number of subsystem types α remains limited. This work ideally extends [12,13], which applied to homogeneous systems (the case of $\alpha = 1$), and although we will focus on discrete-time systems, similar reasonings will work for continuous-time ones as well.

This article is organized as follows. Section 2 introduces the notation and the basic definitions, while Section 3 summarizes the full-block S-procedure. Section 4 contains the main result on the $\mathcal{L}_2$ gain analysis of α-heterogeneous systems, and Section 5 shows how this result can be extended to distributed controller synthesis for such systems. Section 6 shows the application of the synthesis methods to an academic example, and then the conclusions are in Section 7.

2. Preliminaries

2.1. Notation

Let $\mathbb{R}$ be the field of real numbers, $\mathbb{Z}$ the group of integer numbers, and $\mathbb{R}^{n\times m}$ the set of real $n \times m$ matrices. We denote the identity matrix of order n by I_n (or just I if n can be inferred from the context). The notation $A \succ 0$ ($A \prec 0$) indicates that all the eigenvalues of the square matrix $A = A^\top$ are strictly positive (negative). Let $\overline{\sigma}(A)$ denote the maximum singular value of A. We

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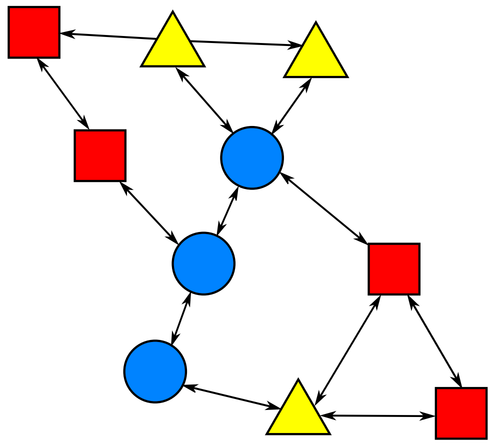

Fig. 1. A heterogeneous system made of the interconnection of subsystems of three different kinds. The arrows represent dynamic interactions among the subsystems.

will also use the symbol $*$ to denote entries that can be inferred from the symmetry of a matrix expression, and the symbol $\star$ in expressions of the type X^TQX to replace X^T and avoid repetitions, i.e. $\star QX = X^TQX$. In this article we make extensive use of the Kronecker product [14], which we denote by the symbol $\otimes$; we remind one of its main properties, according to which:

$$(A \otimes B)(C \otimes D) = (AC) \otimes (BD) \tag{1}$$

if the dimensions of the matrices A, B, C, D are compatible.

2.2. α-heterogeneous systems

We consider a class of systems which we call "α-heterogeneous". Such systems are the result of the interconnection of $N = \sum_{i=1}^{\alpha} N_i$ subsystems of order l; these subsystems belong to α different classes, and N_i elements are present for each class, according to the following definition.

Definition 1 (*α-Heterogeneous Systems*)**.** Let $\mathcal{P}(k)$ be an $N \times N$ matrix, which we call the "pattern matrix", and which can be arbitrarily time-varying. We define $\theta_j = \sum_{i=1}^{j} N_i$ (with $\theta_0 = 0$) and $I_{\{a_1:a_2\}}$ as an $N \times N$ diagonal matrix which contains 1 in the diagonal entries of indices from a_1 to a_2 (included) and 0 elsewhere. Let us consider an Nlth order linear discrete-time dynamical system of equations:

$$\begin{cases} x(k+1) = \mathcal{A}x(k) + \mathcal{B}_w w(k) + \mathcal{B}_u u(k) \\ z(k) = \mathcal{C}_z x(k) + \mathcal{D}_{zw} w(k) + \mathcal{D}_{zu} u(k) \\ y(k) = \mathcal{C}_y x(k) + \mathcal{D}_{yw} w(k) \end{cases} \tag{2}$$

where $k \in \mathbb{Z}$, $x \in \mathbb{R}^{Nl}$ is the state, $u \in \mathbb{R}^{Nm_u}$ is the control input, $w \in \mathbb{R}^{Nm_w}$ is the disturbance, $y \in \mathbb{R}^{Nr_y}$ is the measured output and $z \in \mathbb{R}^{Nr_z}$ is the performance output. We call such a system "α-heterogeneous" (for a given α) iff it has a state space realization with matrices of the kind:

$$\mathcal{M} = \underbrace{\sum_{i=1}^{\alpha} (I_{\{\theta_{i-1}+1:\theta_i\}} \otimes M_a^{(i)})}_{\overline{\mathcal{M}}} + \underbrace{\sum_{i=1}^{\alpha} (I_{\{\theta_{i-1}+1:\theta_i\}} \mathcal{P}(k) \otimes M_b^{(i)})}_{\underline{\mathcal{M}}} \tag{3}$$

where $\mathcal{M}$ represents any of the matrices in (2), and $\mathcal{P}(k)$ is the "pattern matrix"; the matrices $M_a^{(i)}$ are the diagonal blocks of $\mathcal{M}$, while the matrices $M_b^{(i)}$ constitute the off-diagonal blocks, according to the structure of $\mathcal{P}(k)$.

The matrices with superscript "(i)" concern the dynamics of each of the α different kinds of systems. The elements of the state vector in entries from $1 + (i-1)l$ to il, with $1 \leqslant i \leqslant N$ can be considered as the state of the ith subsystem, which belongs to type β if $\theta_{\beta-1} + 1 \leqslant i \leqslant \theta_\beta$. The block diagonal part of the matrices ($\overline{\mathcal{M}}$, made of the submatrices with the subscript "a") represents the internal dynamics of the subsystems, while the part depending on the pattern matrix $\mathcal{P}(k)$ ($\underline{\mathcal{M}}$, made of the submatrices with the subscript "b") accounts for the interactions between subsystems. A sparse pattern matrix indicates that each subsystem interacts only with a limited set of the others, e.g. its neighbors. There is no loss of generality in assuming that all the α different types are of the same order l, or have the same number of input/output channels, as one can add empty rows and columns to upgrade lower order systems to the higher one.

Alpha-heterogeneous systems can be written in a different equivalent form; such observation is reported in the form of a lemma.

Lemma 2. *The system of equations:*

$$\begin{cases} x(k+1) = \overline{\mathcal{A}}x(k) + \overline{\mathcal{B}}_w w(k) + \overline{\mathcal{B}}_u u(k) + \overline{\mathcal{B}}_p p(k) \\ z(k) = \overline{\mathcal{C}}_z x(k) + \overline{\mathcal{D}}_{zw} w(k) + \overline{\mathcal{D}}_{zu} u(k) + \overline{\mathcal{D}}_{zp} p(k) \\ y(k) = \overline{\mathcal{C}}_y x(k) + \overline{\mathcal{D}}_{yw} w(k) + \overline{\mathcal{D}}_{yp} p(k) \\ q(k) = \overline{\mathcal{C}}_q x(k) + \overline{\mathcal{D}}_{qw} w(k) + \overline{\mathcal{D}}_{qu} u(k) \end{cases} \tag{4}$$

where all the matrices are block diagonal, and with $p, q \in \mathbb{R}^{Nm_p}$, and

$$\begin{aligned} \overline{\mathcal{B}}_p &= \sum_{i=1}^{\alpha} I_{\{\theta_{i-1}+1:\theta_i\}} \otimes \begin{bmatrix} A_b^{(i)} & B_{w,b}^{(i)} & B_{u,b}^{(i)} \end{bmatrix} \\ \overline{\mathcal{D}}_{zp} &= \sum_{i=1}^{\alpha} I_{\{\theta_{i-1}+1:\theta_i\}} \otimes \begin{bmatrix} C_{z,b}^{(i)} & D_{zw,b}^{(i)} & D_{zu,b}^{(i)} \end{bmatrix} \\ \overline{\mathcal{D}}_{yp} &= \sum_{i=1}^{\alpha} I_{\{\theta_{i-1}+1:\theta_i\}} \otimes \begin{bmatrix} C_{y,b}^{(i)} & D_{yw,b}^{(i)} & 0 \end{bmatrix} \\ \overline{\mathcal{C}}_q &= \sum_{i=1}^{\alpha} I_{\{\theta_{i-1}+1:\theta_i\}} \otimes \begin{bmatrix} I_l & 0 & 0 \end{bmatrix}^\top \\ \overline{\mathcal{D}}_{qw} &= \sum_{i=1}^{\alpha} I_{\{\theta_{i-1}+1:\theta_i\}} \otimes \begin{bmatrix} 0 & I_{m_w} & 0 \end{bmatrix}^\top \\ \overline{\mathcal{D}}_{qu} &= \sum_{i=1}^{\alpha} I_{\{\theta_{i-1}+1:\theta_i\}} \otimes \begin{bmatrix} 0 & 0 & I_{m_u} \end{bmatrix}^\top, \end{aligned} \tag{5}$$

and $\overline{\mathcal{A}}$, $\overline{\mathcal{B}}_w$, $\overline{\mathcal{B}}_u$, $\overline{\mathcal{C}}_z$, $\overline{\mathcal{D}}_{zw}$, $\overline{\mathcal{D}}_{zu}$, $\overline{\mathcal{C}}_y$, $\overline{\mathcal{D}}_{yw}$ defined according to (3), is equivalent to (2) for

$$p(k) = (\mathcal{P}(k) \otimes I_{m_p})q(k). \tag{6}$$

Proof. Replace the expression of $p(k)$ in (6) into (4), and then simplify the resulting expression using the properties of the Kronecker product. □

Remark 3. As all the matrices in (4) are block diagonal, the interconnections among the different subsystems are only in the relation (6) between $p(k)$ and $q(k)$. Notice also that from now on the symbols $A^{(i)}$, $B_w^{(i)}$, $B_u^{(i)}$, $B_p^{(i)}$, $C_z^{(i)}$, $C_y^{(i)}$, $C_p^{(i)}$, $D_{zw}^{(i)}$, $D_{zu}^{(i)}$, $D_{zp}^{(i)}$, $D_{yw}^{(i)}$, $D_{yp}^{(i)}$ will denote the diagonal blocks of the matrices in (4) corresponding to the ith type of subsystem, which means the blocks between the $(\theta_{i-1}+1)$-th and the θ_i-th (included). So, for example, $A^{(i)} = A_a^{(i)}$, $B_p^{(i)} = \begin{bmatrix} A_b^{(i)} \, B_{w,b}^{(i)} \, B_{u,b}^{(i)} \end{bmatrix}$, and so on.

The system realization defined in (4)–(6), with $m_p = l + r_y + r_u$, is not necessarily minimal, in the sense that many of the entries of the signal $p(k)$ might be identical to zero; this means that the