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Consensus of multiple nonholonomic chained form systems

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ABSTRACT

Consensus problems of multiple nonholonomic systems are considered in this paper. This problem is simplified into consensus problems of two subsystems based on the cascaded structure of nonholonomic chained form systems. Continuous and hybrid distributed controllers have been constructed for these two subsystems respectively based on the theory of cascaded systems. Consensus of multiple nonholonomic chained form systems can be realized using the methodology proposed in this paper no matter whether the group reference signal is persistently exciting or not. Different to previous assumptions on group reference such as persistent excitation or converging to nonzero constant, the condition on the group reference signal have been further relaxed in this paper. Simulation results using Matlab have illustrated the effectiveness of the results presented in this paper.

1. Introduction

1.1. Consensus of multi-agent systems

Multi-agent systems have received a lot of attention in recent years with the great development of computer science, communication, computation and control. Compared with single system, multi-agent systems not only provide robustness to failure and uncertainties but also provide us with some flexibilities in controlling or reconfiguring the whole system. Furthermore, the performance have been greatly leveraged when using multi-agent systems in cooperative surveillance and monitoring, cooperative navigation or cooperative movement. Advantages of using multi-agent system have been shown in unmanned aerial and unmanned underwater vehicles (UAVs and UUVs), unmanned ground vehicles (UGVs), spacecraft formation flight, etc.

Research related to consensus can be dated back to the work of DeGroot [1]. Recent years have seen a lot of works related to this topic. Consensus protocols for single integrator systems have been given in [2–5]. Consensus of double integrator systems have been considered in [3,6,7]. The above consensus results of integrators cannot be directly extended to UAVs, UUVs and UGVs due to systems' complexities. Much more efforts have been put on the cooperative control of multiple nonlinear systems or even nonholonomic systems.

Consensus problem of linear systems with high orders or nonlinear systems is much more challenging compared with that of the integrator systems. For cooperative control of linear time invariant system, a lot of work has adopted the framework of "decomposition" to reduce complexities in the consensus problem. Cooperative control of multiple linear time-invariant (LTI) vehicles have been firstly proposed in [5] based on Schur decomposition where the cooperative control problems of multiple LTI systems have been transformed into control problems of multiple simple subsystems. Then the cooperative control problems of original system can be solved if each subsystem can be stabilized. Inspired by the work of Fax and Murray [5], a general framework of decomposition for the cooperative control of linear time varying systems has been proposed in [8] where distributed controllers have the same interconnection pattern as that among the controlled systems. Results of Fax and Murray [5] and Massioni and Verhaegen [8] have been further extended in [9] to include the case of time-varying topologies and communication delays. The philosophy of "decomposition" is still adopted in [10] for dealing with robust stability of multiple LTI systems under time-varying topologies and communication delays. The framework of "Decomposition" can also be extended to the H_∞ and H_2 cooperative control problem of multiple LTI systems as shown in [11,12].

In order to solve the cooperative control problems of multiple complex dynamic systems, system structure and nonlinear tools have been also introduced in some papers published recently. For example, the cooperative control problems for spatially

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interconnected systems have been considered in [13,14], where distributed controllers with the same structure as that of the spatial systems have been derived using the technique of linear matrix inequality that are amendable to computation. Theories of cascaded systems have been introduced in [15] to solve the attitude synchronization problem of multiple spacecrafts with disturbance. Application of the cascaded theory can also be found in [16] for synchronization of multiple chaotic systems. Nonlinear tools such as the tool dissipative of has been adopted in [17] to characterize the constraints on communication channels in distributed control of dynamic systems. Other nonlinear tools such as ISS in [18,19], internal mode control in [20] and passivity in [21,22] have been introduced in the cooperative control of multiple dynamic systems.

1.2. Distributed control of multiple nonholonomic systems

Compared with linear systems or normal nonlinear systems, control of multiple nonholonomic systems is much more difficult. As shown in [23], there are no smooth (or even continuous) time-invariant state feedback controllers to asymptotically stabilize this kind of systems. Controllers for linear system or normal nonlinear system are not effective any more when being applied to nonholonomic systems. Due to the existence of nonholonomic constraints in mobile robots, unmanned vehicles or underactuated spacecrafts, the cooperative control of multiple nonholonomic systems has attracted a lot of researchers in recent years.

Some previous work on cooperative control of nonholonomic systems have just considered the cooperative control problems when there are some persistent exciting signals in the systems. The nonholonomic robots are required to keep rotating or oscillating in [24] for the cooperative hunting problem. The angle of each nonholonomic mobile robot is left to rotate freely in [25] and the entire group reference's velocity is not allowed to converge to zero in [26,22] for the cooperative path-following problem. For the circular motion control problem considered in [27], assumption of persistently rotating around a virtual beacon has been imposed for each nonholonomic robot. In order to guarantee the controllability of all nonholonomic unicycles, the linear velocity of leader robot is not allowed to equal to zero in [28]. Similar conditions can also be found in [29,30].

The stabilization and tracking control problems of single nonholonomic systems have received a lot of attention. There are many papers about designing controllers for this kind of system using time-varying, discontinuous or even hybrid feedback controllers. It is worthy of considering that whether Brockett's condition is also effective in the cooperative control of multiple nonholonomic systems. It is interesting to note that consensus for networked nonholonomic unicycles has been realized using time invariant continuous feedback controllers in [31] without group reference. Obtained results in [31] have shown that Brockett's condition is not a barrier in the consensus of nonholonomic unicycles. The authors of this paper believe that there are many differences between consensus without reference and consensus with reference for nonholonomic systems. Based on some previous research in [32], the consensus problem with external reference can be decomposed as reference tracking problem and consensus maintaining problem. From this point of view, the consensus protocols obtained in [31] cannot be applied to the consensus problem with external reference that will be considered in this paper.

1.3. Main contribution of this paper

Consensus of multiple nonholonomic chained form systems with desired external reference has been considered in this paper. Structure characters of nonholonomic chained form systems have been embedded in the design of consensus protocols and the consensus problem of nonholonomic chained form systems has been transformed into consensus problems of two simple linear subsystems. Distributed controllers have been given in the end using this framework no matter the group's reference signal converges to zero or not. First difference to [31] lies in that m ($m \geq 3$) dimensional chained form systems have been considered in this paper while only nonholonomic unicycles with $m = 3$ are considered in [31]. Another differences is that consensus on different kinds of external reference signals can be realized using obtained results of this paper. To our best knowledge, there are few papers considering the consensus problem of multiple nonholonomic systems when the group's reference signals converge to zero or equal to zero.

The rest of the paper is organized as follows. System model and problem statements are first given in Section 2. Some preliminary results are also included in Section 2. Our main results on consensus under different external references are presented in Section 3. Section 4 presents simulation results and conclusions are given in Section 5.

2. Preliminaries

Many mechanical systems such as unicycle mobile robots or UAVs with fixed wings can be transformed into chained form by the input and state transformations given in [33]. In this paper, we consider the following widely used canonical chained form of nonholonomic systems

$$\begin{aligned} \dot{x}_{i1} &= u_{i1}, \\ \dot{x}_{i2} &= u_{i2}, \\ \dot{x}_{i3} &= x_{i2}u_{i1}, \\ &\vdots \\ \dot{x}_{im} &= x_{i(m-1)}u_{i1}, \end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, N$ is the number of systems, $u = (u_{i1}, u_{i2})^T$ and $x = (x_{i1}, \ldots, x_{im})^T$ are the control input and state vector of ith mechanical system, respectively. Although the systems represented by (1) are shown to be completely controllable, it cannot be asymptotically stabilized by any smooth state feedback controllers due to Brockett's necessary conditions in [23]. Many controllers and approaches have been proposed to solve the stabilization or tracking control problems for this kind of system such as Lefeber [34], Murray and Sastry [33] and Samson [35]. Based on previous researches on the control of single nonholonomic chained form systems, it is interesting to consider how to solve the cooperative control problem of multiple nonholonomic chained form systems.

Each chained form systems (1) can be represented as a vertices of a graph $G = (\mathcal{V}, E)$, where $\mathcal{V}$ is the set of N vertices, E is the set of all edges in the graph G where one edge $(j, i) \in E$ means that the state information of system j is available to system i. For an undirected graph G with N vertices, the adjacency matrix $A = A(G) = (a_{ij})$ can be defined as $a_{ij} = 1$ if there is one edge $(j, i) \in E$ and $a_{ij} = 0$ otherwise. Let N_i be the collection of neighbors for system i and the desired reference trajectory for the group is described as the following system

$$\begin{aligned} \dot{x}_1^d &= u_1^d, \\ \dot{x}_2^d &= u_2^d, \\ \dot{x}_3^d &= x_2^d u_1^d, \\ &\vdots \\ \dot{x}_m^d &= x_{(m-1)}^d u_1^d, \end{aligned} \tag{2}$$