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On the characterization of the Duhem hysteresis operator with clockwise input–output dynamics

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ABSTRACT

In this paper we investigate the dissipativity property of a certain class of Duhem hysteresis operator, which has clockwise (CW) input–output (I/O) behavior. In particular, we provide sufficient conditions on the Duhem operator such that it is CW and propose an explicit construction of the corresponding function satisfying dissipation inequality of CW systems. The result is used to analyze the stability of a second order system with hysteretic friction which is described by a Dahl model.

1. Introduction

Hysteresis is a common nonlinear phenomenon that is present in diverse physical systems, such as piezo-actuator, ferromagnetic material and mechanical systems. From the perspective of input–output behavior, the hysteretic phenomena can be characterized into counterclockwise (CCW) input–output (I/O) dynamics [1], clockwise (CW) I/O dynamics [2], or even more complex I/O map (such as, butterfly map [3]). For example, backlash operator generates CCW I/O dynamics; elastic–plastic operator generates CW I/O dynamics and Preisach operator can have either CCW or CW I/O dynamics depending on the weight of the hysterons which are used in the Preisach model [4–6]. In the recent work by Angeli [1,7], the counterclockwise (CCW) I/O dynamics of a single-input single-output system is characterized by the following inequality

$$\liminf_{T\to\infty} \int_0^T \dot{y}(t)u(t)\mathrm{d}t > -\infty, \tag{1}$$

where u is the input signal and y is the corresponding output signal. It is assumed that $u \in U$ where U is the set of input signals for which y exists and is well defined for all positive time. Compared with the classical definition of passivity [8,9], it can be interpreted that the system is passive from the input u to the time derivative of the corresponding output y. In particular, (1) holds if there exists a function $H : \mathbb{R}^2 \to \mathbb{R}_+$ such that

$$\frac{\mathrm{d}H(y(t), u(t))}{\mathrm{d}t} \le \dot{y}(t)u(t). \tag{2}$$

Indeed, integrating (2) from 0 to ∞ we obtain (1).

Correspondingly, clockwise (CW) I/O dynamics can be described by the following dissipation inequality

$$\liminf_{T\to\infty} \int_0^T \dot{u}(t)y(t)\mathrm{d}t > -\infty. \tag{3}$$

The notions of counterclockwise (CCW) I/O and clockwise (CW) I/O are also discussed in [10–14].

In our previous results in [15,16], we show that for a certain class of Duhem hysteresis operator $\Phi : u \mapsto \Phi(u, y_0) := y$, we can construct a function $H_{\circlearrowleft} : \mathbb{R}^2 \to \mathbb{R}_+$ which satisfies

$$\frac{\mathrm{d}H_{\circlearrowleft}(y(t), u(t))}{\mathrm{d}t} \le \dot{y}(t)u(t). \tag{4}$$

This inequality immediately implies that such Duhem hysteresis operator is dissipative with respect to the supply rate $\dot{y}(t)u(t)$ and has CCW input–output dynamics. The symbol $\circlearrowleft$ in $H_{\circlearrowleft}$ indicates the counterclockwise behavior of Φ.

In this paper, as a dual extension to [15,16], we focus on the clockwise (CW) hysteresis operator where the supply rate is given by $\dot{u}y$ which is dual to the supply rate $u\dot{y}$ considered in [15,16]. This is motivated by the friction induced hysteresis phenomenon in the mechanical system which has CW I/O behavior from the input relative displacement to the output friction force. One may

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intuitively consider to reverse the input–output relation of the CW hysteresis operator for getting the CCW I/O behavior in the reverse I/O setting. However, this consideration has two drawbacks: (1) the reverse input–output pair may not be physically realizable (this is related to the causality problem in the port-based modeling, such as, the bond graph modeling framework [17]); (2) the operator itself may not be invertible (for example, if the output of the hysteresis operator can be saturated).

In Theorem 1, we provide sufficient conditions on the underlying functions f_1 and f_2 of the Duhem operator, such that it has CW I/O dynamics. Roughly speaking, the functions f_1 and f_2 (as defined later in Section 2) determine two possible different directions (y, u) depending on whether the input u is increasing or decreasing. By evaluating these two functions on two disjoint domains (which are separated by an anhysteresis curve), we can determine whether it has CW I/O dynamics using Theorem 1. This is shown by constructing a function $H_\circlearrowright : \mathbb{R}^2 \to \mathbb{R}_+$ such that the following inequality

$$\frac{\mathrm{d}H_\circlearrowright(y(t), u(t))}{\mathrm{d}t} \leq y(t)\dot{u}(t) \tag{5}$$

holds. The function $H_\circlearrowright$ can also be related to the concept of available storage function from [8,9] where, instead of using the standard supply rate yu, we use the CW supply rate $y\dot{u}$ as shown in Proposition 1 in this paper.

The dissipativity property (5) can be further used in the stability analysis of the systems with CW hysteresis, such as, a second-order mechanical system with hysteretic friction [18] as discussed in Section 4.2. As an illustrative example on the application of (5), let us consider a mechanical system described by

$$m\ddot{x} = F - F_{\text{friction}},$$
$$F_{\text{friction}} = \Phi(x, y_0),$$

with the hysteresis operator Φ satisfying the Dahl model as follows

$$\dot{F}_{\text{friction}} = \rho\left(1 - \frac{F_{\text{friction}}}{F_C}\right)\max\{0, \dot{x}\} + \rho\left(1 + \frac{F_{\text{friction}}}{F_C}\right)\min\{0, \dot{x}\},$$

where m refers to the mass, x refers to the displacement, F is the applied force (control input), $\rho > 0$ describes the stiffness constant, $F_C > 0$ represents the Coulomb friction constant and y_0 is the initial condition of the Dahl model (see, for example, [10]). By taking $x_1 = x$, $x_2 = \dot{x}$ and $x_3 = F_{\text{friction}}$ as the state variables, we can rewrite this hysteretic system into state–space form as follows

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = \frac{F}{m} - \frac{x_3}{m},$$
$$\dot{x}_3 = \rho\left(1 - \frac{x_3}{F_C}\right)\max\{0, x_2\} + \rho\left(1 + \frac{x_3}{F_C}\right)\min\{0, x_2\}.$$

In Section 4.1, we obtain the function $H_\circlearrowright$ satisfying (5) explicitly and it is parameterized by ρ and F_C. Using $V(x_1, x_2, x_3) = \frac{1}{2}mx_2^2 + H_\circlearrowright(x_3, x_1)$ as a Lyapunov function we have

$$\begin{aligned}\dot{V} &= m\dot{x}_2x_2 + \frac{\mathrm{d}H_\circlearrowright(x_3, x_1)}{\mathrm{d}t}\\ &= -x_3x_2 + Fx_2 + \frac{\mathrm{d}H_\circlearrowright(x_3, x_1)}{\mathrm{d}t}\\ &\leq Fx_2.\end{aligned}$$

This inequality establishes that the closed-loop system is passive from the applied force F to the velocity x_2. Thus a simple proportional feedback $F = -dx_2$, where $d > 0$, can guarantee the asymptotic convergence of the velocity x_2 to zero without having to know precisely the parameters ρ and F_C.

2. Duhem operator and clockwise hysteresis operators

Denote $C^1(\mathbb{R}_+)$ the space of continuously differentiable functions $f : \mathbb{R}_+ \to \mathbb{R}$ and $AC(\mathbb{R}_+)$ the space of absolutely continuous functions $f : \mathbb{R}_+ \to \mathbb{R}$. Define $\frac{\mathrm{d}z(t)}{\mathrm{d}t} := \lim_{h\searrow 0^+} \frac{z(t+h)-z(t)}{h}$.

The Duhem operator $\Phi : AC(\mathbb{R}_+) \times \mathbb{R} \to AC(\mathbb{R}_+)$, $(u, y_0) \mapsto \Phi(u, y_0) =: y$ is described by [5,10,19]

$$\dot{y}(t) = f_1(y(t), u(t))\dot{u}_+(t) + f_2(y(t), u(t))\dot{u}_-(t), \quad y(0) = y_0, \tag{6}$$

where $\dot{u}_+(t) := \max\{0, \dot{u}(t)\}$, $\dot{u}_-(t) := \min\{0, \dot{u}(t)\}$. The functions f_1 and f_2 are assumed to be C^1.

The existence of solutions to (6) has been reviewed in [5]. In particular, if for every $\xi \in \mathbb{R}$, f_1 and f_2 satisfy

$$\begin{aligned}&(\sigma_1 - \sigma_2)[f_1(\sigma_1, \xi) - f_1(\sigma_2, \xi)] \leq \lambda_1(\xi)(\sigma_1 - \sigma_2)^2,\\ &(\sigma_1 - \sigma_2)[f_2(\sigma_1, \xi) - f_2(\sigma_2, \xi)] \geq -\lambda_2(\xi)(\sigma_1 - \sigma_2)^2,\end{aligned} \tag{7}$$

for all $\sigma_1, \sigma_2 \in \mathbb{R}$, where λ_1 and λ_2 are nonnegative, then the solution to (6) exist and Φ maps $AC(\mathbb{R}_+) \times \mathbb{R} \to AC(\mathbb{R}_+)$. We will assume throughout the paper that the solution to (6) exists for all $u \in AC(\mathbb{R}_+)$ and $y_0 \in \mathbb{R}$.

As a dual definition to counterclockwise (CCW) I/O behavior [1], we define the clockwise (CW) I/O dynamics as follows

Definition 1. An operator Q is *clockwise* (CW) if for every $u \in U$ with the corresponding output map $y := Qu$, where U is the space of input signals such that y is well-defined for all positive time, the following inequality holds

$$\liminf_{T\to\infty} \int_0^T y(t)\dot{u}(t)\mathrm{d}t > -\infty. \tag{8}$$

For the Duhem operator Φ, inequality (8) holds if there exists a function $H_\circlearrowright : \mathbb{R}^2 \to \mathbb{R}_+$ such that for every $u \in AC(\mathbb{R}_+)$ and $y_0 \in \mathbb{R}$, the inequality

$$\frac{\mathrm{d}H_\circlearrowright(y(t), u(t))}{\mathrm{d}t} \leq y(t)\dot{u}(t), \tag{9}$$

holds for all t where $y := \Phi(u, y_0)$.

In the following subsections, we describe several well-known hysteresis operators which generate clockwise I/O dynamics and we recast these operators into the Duhem operator as in (6).

2.1. Dahl model

The Dahl model [20,21] is commonly used in mechanical systems, which represents the friction force with respect to the relative displacement between two surfaces in contact. The general representation of the Dahl model is given by

$$\dot{y}(t) = \rho\left|1 - \frac{y(t)}{F_c}\mathrm{sgn}(\dot{u}(t))\right|^r \mathrm{sgn}\left(1 - \frac{y(t)}{F_c}\mathrm{sgn}(\dot{u}(t))\right)\dot{u}(t), \tag{10}$$

where y denotes the friction force, u denotes the relative displacement, $F_c > 0$ denotes the Coulomb friction force, $\rho > 0$ denotes the rest stiffness and $r \geq 1$ is a parameter that determines the shape of the hysteresis loops.

The Dahl model can be described by the Duhem hysteresis operator (6) with

$$f_1(\sigma, \xi) = \rho\left|1 - \frac{\sigma}{F_c}\right|^r \mathrm{sgn}\left(1 - \frac{\sigma}{F_c}\right), \tag{11}$$

$$f_2(\sigma, \xi) = \rho\left|1 + \frac{\sigma}{F_c}\right|^r \mathrm{sgn}\left(1 + \frac{\sigma}{F_c}\right). \tag{12}$$