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Learning from neural control of nonlinear systems in normal form[☆]

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ABSTRACT

A deterministic learning theory was recently proposed which states that an appropriately designed adaptive neural controller can learn the system internal dynamics while attempting to control a class of simple nonlinear systems. In this paper, we investigate deterministic learning from adaptive neural control (ANC) of a class of nonlinear systems in normal form with unknown affine terms. The existence of the unknown affine terms makes it difficult to achieve learning by using previous methods. To overcome the difficulties, firstly, an extension of a recent result is presented on stability analysis of linear time-varying (LTV) systems. Then, with a state transformation, the closed-loop control system is transformed into a LTV form for which exponential stability can be guaranteed when a partial persistent excitation (PE) condition is satisfied. Accurate approximation of the closed-loop control system dynamics is achieved in a local region along a recurrent orbit of closed-loop signals. Consequently, learning of control system dynamics (i.e. closed-loop identification) from adaptive neural control of nonlinear systems with unknown affine terms is implemented.

1. Introduction

Adaptive neural control has recently attracted tremendous interest in both control theory and practical applications [1–7,24,26]. However, the problem of whether the neural networks (NNs) employed in adaptive neural controllers indeed implement their function approximation ability has been less investigated. As a consequence, most of the adaptive neural controllers have to recalculate the control parameters even for repeating the same control task.

Motivated by human abilities of "learning by doing" and "doing with learned knowledge", recently, a deterministic learning mechanism was presented in [8], by which an adaptive neural controller is capable of learning the unknown dynamics in a closed-loop control process. A simple nonlinear system with an unity control gain was considered in [8]. The deterministic learning is achieved according to the following elements: (i) tracking control of the system states to a recurrent reference orbit; (ii) satisfaction of a partial persistent excitation (PE) condition by the localized RBF network; (iii) exponential stability of the closed-loop system along the tracking orbit, including the convergence of certain neural weights to their optimal values; and (iv) accurate approximation of the unknown dynamics in a local region along the recurrent tracking orbit. A neural learning control scheme was also proposed which can effectively utilize the learned knowledge for improved control performance. The deterministic learning approach provides a simple and efficient solution to the problem of learning and control for dynamical closed-loop systems.

In this paper, we investigate deterministic learning from neural control of a general class of nonlinear systems in normal form with unknown affine terms. In the literature of nonlinear control, it is well known that systems with affine terms are more difficult to be dealt with. From the perspective of learning, it will be shown that the existence of affine terms also leads to difficulties which prevent the occurrence of learning (i.e. accurate parameter convergence) in the adaptive neural control process. Therefore, to make the deterministic learning control more practical on the basis of [8], it is necessary to investigate how to achieve deterministic learning for nonlinear systems with unknown affine terms.

Through necessary transformation, a model reference adaptive (neural) control system can be considered as a perturbed linear time-varying (LTV) system in the following form:

$$\begin{bmatrix} \dot{e} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} A(t) & B^{T}(t) \\ -C(t) & 0 \end{bmatrix} \begin{bmatrix} e \\ \theta \end{bmatrix} + \delta(t) \tag{1}$$

where $e \in R^n$ denotes the state tracking errors, $\theta \in R^p$ denotes the parameter estimating errors, $A(t) \in R^{n\times n}$, $B(t) \in R^{p\times n}$,

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$C(t) \in R^{p\times n}$, and $\delta(t) \in R^{(n+p)}$ represents bounded disturbances caused by the NN approximation errors. For deterministic learning from the simple nonlinear systems with a unit control gain, $A(t)$ in (1) is time invariant [8]. Stability and convergence of such kind of LTV systems have been discussed in several works (e.g., [9–11,25]). For deterministic learning from nonlinear systems in normal form, however, one difficulty lies in that the unknown affine term will appear in the closed-loop adaptive system so that $A(t)$ in (1) is a time-varying term. The stability analysis of this kind of LTV systems cannot be handled by conventional results of adaptive systems. Recently, this kind of LTV systems was revisited in [12] and a new method was provided. Nonetheless, it is still inapplicable to deterministic learning without necessary extension. Another difficulty lies in that with the affine terms appeared in the closed-loop adaptive system, it may amplify the NN approximation errors and lead to large $\delta(t)$. This will prevent the occurrence of learning even when exponential stability of the nominal part of the closed-loop adaptive system is achieved.

In this paper, the difficulties concerning deterministic learning for nonlinear systems with unknown affine terms will be resolved as follows. Firstly, an extension of the result on stability of a class of LTV systems [12] is presented. It is shown that with the satisfaction of a partial PE condition and with some other mild conditions, exponential stability of this class of LTV systems can be achieved. Secondly, to overcome the difficulties caused by the unknown affine terms, a state transformation is introduced which turns the closed-loop adaptive system into the form of the perturbed LTV system with small perturbations. Exponential stability of the perturbed LTV system is obtained, and convergence of partial neural weights is guaranteed. Accurate approximation of the closed-loop control system dynamics is achieved in a local region along a recurrent orbit of closed-loop signals. Consequently, learning of control system dynamics (i.e. closed-loop identification) from adaptive neural control of nonlinear systems with unknown affine terms is implemented. This result will be useful for further research on learning from more general nonlinear systems (such as strict-feedback systems and pure-feedback systems), and is applicable to many industrial applications.

The rest of the paper is organized as follows: Section 2 presents problem formulation and preliminary results. Stability analysis of a class of LTV systems is presented in Section 3. Learning from adaptive neural control of nonlinear systems in normal form is presented in Section 4. Section 5 contains the conclusion.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider the following nonlinear system in normal form

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad 1 \le i \le n-1 \\ \vdots \\ \dot{x}_n = f(x) + g(x)u, \quad n \ge 2 \end{cases} \tag{2}$$
$$y = x_1$$

where $x = [x_1, \dots, x_n]^{\mathrm{T}} \in R^n$, $u \in R$ and $y \in R$ are the state variables, the system input and output, respectively, both $f(x)$ and the affine term $g(x)$ are unknown smooth nonlinear functions.

The following reference model is considered:

$$\begin{cases} \dot{x}_{di} = x_{di+1}, \quad i = 1, \dots, n-1 \\ \dot{x}_{dn} = f_d(x_d, t) \end{cases} \tag{3}$$
$$y_d = x_{d1}$$

where $x_d = [x_{d1}, \dots, x_{dn}]^{\mathrm{T}} \in R^n$ is the system state, y_d is the output and $f_d(\cdot)$ is a known smooth nonlinear function. The system orbit starting from the initial condition $x_d(0)$ is denoted as φ_d. Assume that the states of the reference model be uniformly bounded, i.e., $x_d(t) \in \Omega_d$, $\forall t \ge 0$, and the system orbit φ_d be a recurrent motion.[1]

The objective is to develop an adaptive neural controller using a localized RBF network such that the output y follows the desired trajectory y_d generated from the reference model, and accurate NN approximation (learning) of the closed-loop control system dynamics is achieved in a local region along an orbit of closed-loop signals.

2.2. Localized RBF networks

The RBF networks can be described by $f_{nn}(Z) = \sum_{i=1}^{N} w_i s_i(Z) = W^T S(Z)$ [14], where $Z \in \Omega_Z \subset R^q$ is the input vector, $W = [w_1, \dots, w_N]^{\mathrm{T}} \in R^N$ is the weight vector, N is the NN node number, and $S(Z) = [s_1(\|Z-\xi_1\|), \dots, s_N(\|Z-\xi_N\|)]^{\mathrm{T}}$, with $s_i(\cdot)$ being a radial basis function, and ξ_i $(i = 1, \dots, N)$ being distinct points in state space. The Gaussian function $s_i(\|Z-\xi_i\|) = \exp\left[\frac{-(Z-\xi_i)^T(Z-\xi_i)}{\eta_i^2}\right]$ is one of the most commonly used radial basis functions, where $\xi_i = [\xi_{i1}, \xi_{i2}, \dots, \xi_{iq}]^{\mathrm{T}}$ is the center of the receptive field and η_i is the width of the receptive field. The Gaussian function belongs to the class of localized radial basis functions in the sense that $s_i(\|Z-\xi_i\|) \to 0$ as $\|Z\| \to \infty$.

It has been shown in [14,15] that for any continuous function $f(Z): \Omega_Z \to R$ where $\Omega_Z \subset R^q$ is a compact set, and for the NN approximator, where the node number N is sufficiently large, there exists an ideal constant weight vector W^*, such that for each $\epsilon^* > 0$, $f(Z) = W^{*T}S(Z) + \epsilon(Z)$, $\forall Z \in \Omega_Z$, where $|\epsilon(Z)| < \epsilon^*$ is the approximation error. Moreover, for any bounded trajectory $Z_\zeta(t)$ within the compact set Ω_Z, $f(Z)$ can be approximated by using a limited number of neurons located in a local region along the trajectory: $f(Z) = W_\zeta^{*T} S_\zeta(Z) + \epsilon_\zeta$, where $S_\zeta(Z) = [s_{j_1}(Z), \dots, s_{j_\zeta}(Z)]^{\mathrm{T}} \in R^{N_\zeta}$, with $N_\zeta < N$, $|s_{j_i}| > \iota$ $(j_i = j_1, \dots, j_\zeta)$, $\iota > 0$ is a small positive constant, $W_\zeta^* = [w_{j_1}^*, \dots, w_{j_\zeta}^*]^{\mathrm{T}}$, and ϵ_ζ is the approximation error, with $\big||\epsilon_\zeta| - |\epsilon|\big|$ being small.

Based on previous results on the PE property of RBF networks [16,17], it is shown in [8,18] that for a localized RBF network $W^{\mathrm{T}}S(Z)$ whose centers are placed on a regular lattice, almost any recurrent trajectory $Z(t)$ can lead to the satisfaction of the PE condition of the regressor subvector $S_\zeta(Z)$.

3. Stability of a class of LTV systems

In this section, we study the exponential stability of a class of LTV systems associated with adaptive neural control of nonlinear systems with unknown affine terms.

In [12], under some mild assumptions, sufficient and necessary conditions for exponential stability of system (1) with $A(t)$ time varying and $\delta(t) = 0$ was presented.

Assumption 3.1 (*[12]*). There exists a $\phi_M > 0$ such that, for all $t \ge 0$, the following bound is satisfied

$$\max\left\{\|B(t)\|, \left\|\frac{\mathrm{d}B(t)}{\mathrm{d}t}\right\|\right\} \le \phi_M \tag{4}$$

Assumption 3.2 (*[12]*). There exist symmetric matrices $P(t)$ and $Q(t)$ such that $P(t)B(t) = C(t)$ and $-Q(t) = A^{\mathrm{T}}(t)P(t) +$

[1] The recurrent motions comprise the most important types (though not all types) of trajectories generated from nonlinear dynamical systems, including periodic, quasi-periodic, almost-periodic and even chaotic trajectories (see [13] for a rigorous definition of recurrent trajectory).