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On controller & capacity allocation co-design for networked control systems[☆]

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ABSTRACT

This paper presents a framework for examining joint optimal channel-capacity allocation and controller design for networked control systems using store-and-forward networks in a discrete-time linear time-invariant setting. The resultant framework provides a synthesis procedure for designing distributed linear control laws for capacity-constrained networks taking the allocation of the capacity within the network into account.

For networked systems with a large number of nodes (sensors, controllers and actuators) or systems where mutual communication amongst all nodes is difficult or slow, networked control system design presents significant challenges including considerations of the choice of network topology (inter-node connectivity and inter-node delay), allocation of inter-node channel capacity and total network capacity in addition to the controller design under these communication constraints. These considerations are similar to the guiding principles of *distributed* control system design.

The philosophy of distributed system design is to dispense with the notion that the system in question can be controlled through a single centralized control law and distribute the control task across a number of communicating controllers that, together, achieve the desired behavior. Our focus will be on stabilization.

Communication over networks entails a cost in the form of delays and capacity requirements needed to achieve stability. In various settings, the results presented in [1–4] have established the minimum data rates (channel capacities) needed to stabilize linear systems.[1] For discrete-time systems, both Braslavsky et al. [1] and Nair et al. [2] establish that the minimum data rate needed to stabilize a linear plant of the form

$$x^{+} = Ax + Bu \qquad y = Cx \tag{1}$$

is given by $R > C$, where

$$C = \sum_{|\eta_i|\geq 1} \log_2 |\eta_i| \text{ bits per second} \tag{2}$$

and η_i are the eigenvalues of A.

It was shown that (2) is necessary and sufficient for the existence of a coding and control law that gives exponential convergence of the state to the origin from a random initial state. The primary observation in [1] was that the channel may impose a bit rate limitation for signals in the control loop through a constraint on the signal-to-noise ratio (SNR) for the communication channel. SNR-constrained channels were considered [1] and all pre- and post-signal processing involved in the communication link was restricted to LTI filtering and digital-to-analog and analog-to-digital type operations. Hence, the communication link reduces to the noisy channel itself. By application of the Shannon–Hartley Theorem, [1] recovers the bound (2) for discrete-time systems and presents analogous bounds for continuous-time systems.

From an NCS point of view, these results are phrased in terms of the information flow from a controller to a monolithic actuator and not the information flows between nodes (which directly measure plant states or outputs) and controllers. Using an SNR characterization of channel capacity, this paper will present a discrete-time framework for optimal distributed (state-feedback) controller design specifically taking link-to-link capacity and network structure into account.

The signal in the per-channel SNR ratio is the state x_i and we assume the (local) controller directly actuates the plant, hence there are no capacity constraints in communicating the control signal to the plant. Our design framework establishes

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1 Nonlinear results are established in [5,3] but we restrict our attention to linear results where the bounds have been shown to be necessary and sufficient and the calculations of the bounds are tractable.

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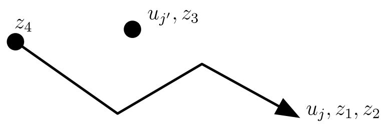

Fig. 1. Directed graph representation of an information structure.

the lowest SNR needed to communicate individual states to *any controller* u_j but without reference to the path traversed in the network; the relevant SNR, and hence, capacity, for each state is maintained along path components from the node measuring the state to (each) controller. The next phase of the design process is solving for the optimal allocation of the capacities within directed graphs. Regarding the node measuring state x_r as a producer of a commodity r and each controller dependent on x_r as a consumer, the capacity allocation problem can be posed as a multi-commodity graph flow problem that is solved efficiently through linear programming (LP).

The design process posited entails:

(a) solving for the optimal linear control law compatible with the information structure imposed by the network graph;
(b) solving for end-to-end (state-to-controller) capacities; and
(c) determining the optimal allocation of capacities within the network, thereby establishing the capacities along each edge of directed graphs describing the network.

1. Information structures for control

An information structure for a control system is, essentially, a description of the dependencies of the control law on observable system information. Henceforth, our discussion will be restricted to stochastic discrete-time systems with state feedback:

$$z^+ = \Phi z + \Gamma u + w, \tag{3}$$

and cost functions of the form

$$v = \mathbf{E}|Cz + Du|^2, \tag{4}$$

where $z(0)$ is Gaussian and zero-mean and w is zero-mean white Gaussian noise with unit variance (identity covariance), hence, "dependence" is equivalent to non-zero covariance between respective random variables. As an example of an information structure, neglecting channel noise and writing $u_j(k) = \mu_j(z_1(k), z_2(k), z_4(k-3))$, we are implying that control u_j depends only the current values of z_1, z_2 and z_4 delayed by three time-steps (and no other random variables). We can also express this information structure in the form of a directed graph as in Fig. 1.

Dependency of $u_j(k)$ on state $z_i(k - \Delta_{ji})$ in an information structure is equivalent to connectivity along a directed path in the graph with a path-length of Δ_{ji}. Note that dependencies of $u_j(k)$ on $z_i(k)$ corresponds to collocation of u_j and z_i at a graph vertex and independence of u_j and z_i is equivalent to asserting that there does not exist a directed path from z_i to u_j.

Suppose now that instead of representing an information structure, Fig. 1 represents a network topology. That is, the directed paths represent existence of a directed *network* communication channel between elements of the path (with delays commensurate with path-length). In a directed sense, each connected component of the graph is a representation of *allowable* dependencies of controllers on incident state variables in that graph component.

In the discussion of information structures and network-connectivity, it becomes evident that the plant itself may be used as a communication channel leading to the possibility of complex nonlinear control laws that "signal" information through the plant. The optimal design problem may quickly become intractable if the admissible information structures are not carefully characterized, as demonstrated by the so-called Witsenhausen counterexample in stochastic optimal control [6] where it was shown that the optimal strategies *may not be linear* in spite of the apparent simplicity of the problem and Gaussian signals. Notwithstanding nonlinear laws proposed in [7] and the references cited therein, progress towards tractability of the synthesis problem can be made by eliminating information structures that create signaling incentives. Progress has been made in rendering the synthesis problem tractable and, in particular, convex in [8–12] in various settings.

Our presentation builds on that of [12] where the (infinite-horizon) distributed control synthesis problem is solved by expressing information structure constraints as covariance constraints and restricting the set of admissible information structures to those that eliminate the "signaling incentive". The synthesis problem is shown to be convex and solutions can efficiently be obtained by solving a linear matrix inequality (LMI); e.g., through interior-point methods described in [13]. The two central observations in [12] are that for systems of the form (3) employing static state feedback, we can express information structure constraints as controller-process noise covariance constraints; and, for each (sub)controller u_j, either the network propagates information at least as fast as the plant does, or, the network propagates information that is not communicable through the plant. With these observations, the optimal synthesis problem for the given information structure becomes

$$\begin{aligned} &\text{minimize } \mathbf{E}|Cz + Du|^2 \quad \text{subject to} \\ &\qquad \mathbf{E}\{x(k)^{\mathrm{T}} R_j u(k)\} = 0 \quad 1 \le j \le J \end{aligned}$$

for appropriately chosen matrices R_j and $x = \left[z(k)^{\mathrm{T}}\ w(k-1)^{\mathrm{T}}\ \dots\ w(k-N)^{\mathrm{T}}\right]^{\mathrm{T}}$ and $u(k) = \left[u_1(k)^{\mathrm{T}}\ \dots\ u_J(k)^{\mathrm{T}}\right]^{\mathrm{T}}$, where N is the length of the largest delay from any measurement to any subcontroller. By careful selection of a cost function that penalizes per-state SNR and decoupling the notion of an information structure from that of network-connectivity, the proceeding section establishes the basic result for the optimal capacity-control co-design framework of this paper.

2. State feedback with covariance constraints

The following result is a general theorem on optimal state-feedback design with covariance constraints for systems of the form $x^+ = Ax + Bu + Fw$, where state variables are subject to additive noises and is analogous to [12, Theorem 1] with three key differences:

1. the control law is the optimal *linear* law i.e., we assume linearity whereas linearity is proved in [12];
2. state variables are subject to additive white Gaussian channel-noise e in the feedback path (as distinct from the usual process noise w) and as the solution of the optimization problem establishes the respective cross and auto-covariances of w, x, e, u, the per-state SNRs
$$\lambda_i = \frac{\mathbf{E}\{x_i^2\}}{\mathbf{E}\{e_i^2\}} \tag{5}$$
are easily found; and
3. the cost function now includes a term that is SNR-dependent i.e., dependent on $\lambda = \mathrm{diag}\{\lambda_1, \dots, \lambda_{n_x}\}$.

Theorem 2.1. *Suppose* $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times m_x}$, $C \in \mathbb{R}^{p_x \times n_x}$, $D \in \mathbb{R}^{p_x \times m_x}$ *and* $R_j \in \mathbb{R}^{n_x \times m_x}$ *for* $1 \le j \le J$. *Then for every* $\delta \ge 0$, *the following two statements are equivalent:*