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Systems & Control Letters 57 (2008) 626–630
systems & control letters
www.elsevier.com/locate/sysconle

Input–output feedback linearization for nonminimum phase nonlinear systems through periodic use of synthetic outputs

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Received 28 May 2007; received in revised form 17 December 2007; accepted 16 January 2008
Available online 11 March 2008

Abstract

Input–output feedback linearization provides a convenient means of extending linear control strategies such as output zeroing or pole placement to the case of nonlinear affine in the input systems, but such extensions cannot be applied in the presence of nonminimum phase characteristics. This paper overcomes this difficulty through the periodic use of a finite number of synthetic outputs which are so constructed as to define embedded dynamics with stable zero dynamics. The efficacy of the method is demonstrated by means of a numerical example.

Keywords: Input–output feedback linearization; Nonminimum phase; Synthetic outputs; Zero dynamics; Periodic output zeroing

1. Introduction

Input–output feedback linearization (IOFL) provides an appealing means for achieving regulation of setpoint tracking for single-input single-output (SISO) nonlinear systems which are affine in the input [1]. However the presence of nonminimum phase characteristics prevents the use of IOFL, and, to overcome this, previous work has either resorted to the use of minimum phase approximations [2,3], or the use of a synthetic output [4–6], or the combined use of a synthetic output and resetting [7]. This paper instead considers the offline definition of $\nu - 1$ synthetic outputs, where ν is some positive integer, which together with the actual system output are to be used periodically. A discrete time IOFL control law is defined at successive sampling instants on the basis of the actual system output and each synthetic output in turn, and this cycle is to be repeated every ν samples. The idea here is to construct the synthetic outputs so that the embedded dynamics governing the actual system output at intervals of ν sampling instants are characterised by stable zero dynamics, despite the zero dynamics of the original system being unstable. Using this technique, we define a locally stabilizing control strategy which achieves output zeroing with respect to the actual output (albeit at every ν steps); earlier approaches [4–7] deployed a regulation strategy on synthetic outputs. The efficacy of this is illustrated through simulations carried out on the model of a grain dryer [8].

2. Problem formulation

Consider a SISO input affine nonlinear system with state space representation

$$x(t+1) = f(x(t)) + g(x(t))u(t) \\ y_0(t) = C_0 x(t), \tag{1}$$

where $x \in \mathfrak{R}^n$, and f, g are two n-dimensional vector functions of x which are assumed to be continuous at the origin. Without loss of generality, rather than considering the setpoint tracking problem, here consideration is given to the problem of output regulation. Under the assumption (which is made for convenience and without loss of generality) that in a neighbourhood of the origin (1) has relative degree 1, the optimal regulation strategy (for the case of no penalty on control activity) is the Output Zeroing (OZ) control law[1]:

[1] In the case of relative degrees higher than 1, the output zeroing control move cannot be given in closed form and has to be computed numerically.

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0167-6911/$ - see front matter
doi:10.1016/j.sysconle.2008.01.004

$$u(t) = -\frac{C_0 f(t)}{C_0 g(t)} \tag{2}$$

since this would make the output zero in one control move and would keep it there subsequently. This control law however would fail to stabilize the origin if the zero dynamics of (1) about the origin were unstable. To circumvent this difficulty, the current paper proposes the periodic use of OZ control moves. The first of these is given by (2), and the remainder within each periodic cycle are defined as OZ moves based on a number of pre-determined synthetic outputs, defined as:

$$y_i = C_i x, \quad i = 1, \dots, \nu - 1. \tag{3}$$

In the context of the development below, the choice of synthetic outputs that depend linearly on the state vector makes good sense; it is noted here that the same linearity assumption has been made with respect to the actual output, but this is done purely for convenience and does not affect the results of the paper which depend on parameters obtained by a process of Jacobian linearization. For convenience use will be made of the notation

$$z(t) = z(k\nu + j) = z_{k,j}, \tag{4}$$

where k assumes the values $0, 1, \dots$ and for each such value j increments through the sequence $\{0, 1, \dots, \nu - 1\}$. Accordingly the periodic OZ strategy becomes:

$$u_{k,j} = -\frac{C_j f(x_{k,j})}{C_j g(x_{k,j})} \tag{5}$$

and thus the first move of each periodic cycle coincides with (2), which as mentioned above fails to stabilize the origin. To compensate for this, here the synthetic outputs are chosen so that the embedded dynamics from $t = k\nu$ to $k\nu + j$ are minimum phase for all $j^* \le j$, for some integer value $0 < j^* < \nu$. The term minimum phase is used here to indicate that the zero dynamics are stable, a condition which is guaranteed by the stability of the Jacobian linearization of the zero dynamics. The following definition will be needed in the statement of the result below:

$$A = \left.\frac{\partial f}{\partial x}\right|_{x=0}; \qquad B = g|_{x=0}. \tag{6}$$

$N \in \mathbb{R}^{n-1 \times n}$ is a full rank matrix representation of the left null space of B satisfying

$$NB = 0 \tag{7}$$

and $M_i \in \mathbb{R}^{n \times n-1}$ is a full rank matrix representation of the kernel of C_i satisfying

$$C_i M_i = 0, \qquad NM_i = I_{n-1}, \quad i = 0, \dots \nu - 1. \tag{8}$$

Assumption 2.1. There is a neighbourhood of the origin where (1) has definite relative degree for all C_i; for convenience this relative degree will be taken to be 1, it being understood that extensions to the general case are straightforward.

Theorem 2.1. *The Jacobian linearization of the embedded zero dynamics from $t = k\nu$ to $k\nu + j$ of (1) under (5) is stable if and only if the eigenvalues of*

$$\Phi_{0\to j} = \prod_{i=0}^{j} \mathrm{NAM}_i \tag{9}$$

are less than 1 in modulus.

Proof. Under (5) the evolution of the state from $k\nu + i$ to $k\nu + i + 1$ is given by

$$x_{k,i+1} = f(x_{k,i}) - g(x_{k,i})\frac{C_i f(x_{k,i})}{C_i g(x_{k,i})} = h_{i+1}(x_{k,i}) \tag{10}$$

so that the embedded dynamics from $k\nu$ to $k\nu + j$ are given by

$$x_{k,j} = h_{k,j}(h_{k,j-1}(\dots h_{k+1}(x_{k,0})\dots)) = H_j(x_{k,0}). \tag{11}$$

From (10), given that the control move of (5) tends to zero as $x_{k,i} \to 0$ it follows that

$$\left.\frac{\partial h_{i+1}(x_{k,i})}{\partial x_{k,i}}\right|_{x_{k,i}=0} = \left.\frac{\partial f(x_{k,i})}{\partial x_{k,i}}\right|_{x_{k,i}=0} - \left.\left[g(x_{k,i})\frac{\partial}{\partial x_{k,i}}\left(\frac{C_i f(x_{k,i})}{C_i g(x_{k,i})}\right)\right]\right|_{x_{k,i}=0} = A - B\frac{C_i A}{C_i B}. \tag{12}$$

Hence differentiating the composition of transition maps, H_j, gives

$$\frac{\partial x_{k,j}}{\partial x_{k,0}} = \prod_{i=0}^{j} \frac{\partial x_{k,i+1}}{\partial x_{k,i}} = \prod_{i=0}^{j}\left(A - B\frac{C_i A}{C_i B}\right). \tag{13}$$

By definition, the zero dynamics under consideration maintain the synthetic outputs at zero, or equivalently ensure that

$$x_{k,i} = M_i \beta_{k,i} \tag{14}$$

which is automatically satisfied under (5), provided that the state at the start of the periodic cycle itself satisfies (14). Substitution of (14) into (11) and pre-multiplication by N gives a nonlinear expression for the evolution of β which of course describes the zero dynamics:

$$\beta_{k,j} = Nx_{k,j} = NH_j(x_{k,0}) = NH_j(M_0\beta_{k,0}). \tag{15}$$

Differentiating this expression and using (12) gives the state matrix for the linearized dynamics as

$$\frac{\partial \beta_{k,j}}{\partial \beta_{k,0}} = N\prod_{i=0}^{j}\frac{\partial x_{k,i+1}}{\partial x_{k,i}} = N\prod_{i=0}^{j}\left(A - B\frac{C_i A}{C_i B}\right)M_0. \tag{16}$$

The result of the theorem follows from the identity

$$I - \frac{BC_i}{CB_i} = M_i N. \quad \blacksquare \tag{17}$$

Corollary 2.2. *The minimum phase property of Theorem 2.1 can be achieved if and only if the pair*

$$\left[\Phi_{0\to j-1}\mathrm{NAN}^{\dagger}\,\Phi_{0\to j-1}\mathrm{NAB}\right] \tag{18}$$

is controllable, where $N^{\dagger}$ denotes a left inverse of N.