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Robust H_∞ output-feedback control of systems with time-delay☆

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Abstract

The problem of designing robust dynamic output-feedback controllers for linear, continuous, time-invariant systems with uncertain time-delay in the measured output and/or the control input and with polytopic type parameter uncertainties is considered. Given a transfer function matrix of a system with uncertain real parameters that reside in some known ranges, an appropriate, not necessarily minimal, state–space model of the system is described which permits reconstruction of its states. The resulting retarded model incorporates the uncertain parameters of the transfer function matrix in the state–space matrices and the uncertain time-delay that occurs in the control channel. To this model, the recent theory of robust H_∞ state-feedback control for retarded systems is applied. The theory is used to solve a benchmark problem of distillation column robust control design.

Keywords: Robust control H_∞; Time delay; Output feedback; LMI; Polytopic uncertainty; Distillation column

1. Introduction

One of the main contributions of H_∞ control theory is the ability to construct controllers and estimators that are robust in the sense that they are built to cope with prescribed ranges of parameter uncertainties. Applying the method of convex programming, relatively simple solutions have been obtained to the problems of state-feedback stabilizing and disturbance attenuation in linear systems with polytopic or norm-bounded uncertainties. Unfortunately, it has soon been realized that the corresponding output-feedback control problems are not convex [4], and various iterative methods have been suggested to derive the required controllers [4]. These methods are known to converge locally and they do not necessarily achieve a global minimum for the disturbance attenuation level.

This obstacle has also been encountered in the design of output controllers in cases where a time-delay appears in the control channel. In the case where the parameters of the system are all known, a solution to the H_∞ control problem was achieved by [8]. This solution, however, does not cope with parameter uncertainties and it requires the knowledge of the delay. On the other hand, a comprehensive treatment of systems with time-delay has been made based on the Lyapunov–Krasovskii approach [1]. While the latter provides a means for dealing with parameter uncertainties and constant unknown time-delay it entails an overdesign that stems from reducing a problem which is of infinite dimensions in nature to one that is of finite dimensions. In the presence of parameters uncertainties, these latter methods are suitable for designing state-feedback controllers; unfortunately, however, they cannot guarantee stability and performance when applying output-feedback control.

Recently, a method has been proposed for the design of robust output-feedback controllers for discrete-time systems without delay but with polytopic type uncertainties [10]. An augmented state–space model for the nonretarded system is first constructed which includes delayed versions of the system inputs and outputs. On this model robust state-feedback controllers are applied which satisfy the performance requirements. These controllers are then translated into equivalent dynamic output-feedback controllers. A similar approach is adopted in the present paper for the continuous-time case with uncertain delay in the control channel. Also here, an augmented state–space model is achieved to which a state-feedback control corresponds to a dynamic output-feedback controller that is applied to the original system. It is shown that the uncertain delay in the control channel can be translated to a state-delay

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in the augmented state–space description. The recent result on the robust state-feedback stabilization and control of systems with state-delays of [9] are then applied to obtain the required output-feedback controller.

The new theory is used to solve the benchmark problem of robust control design of distillation columns [6]. This problem has been studied by several authors, see [7] and references therein. It is shown how an application of a simple PI controller and a simple notch prefilter satisfies the design specifications, in spite of large parameter uncertainty and the uncertain delay in the control channel.

Notation: Throughout the paper, the superscript 'T' stands for matrix transposition, $\Re^n$ denotes the n dimensional Euclidean space with vector norm $|\cdot|$, $\Re^{n\times m}$ is the set of all $n \times m$ real matrices, and the notation $P>0$, for $P \in \Re^{n\times n}$ means that P is symmetric and positive definite. The space of square integrable functions over $[0\ \ \infty)$ is denoted by $\mathscr{L}_2$, and $\|\cdot\|_2$ stands for the standard $\mathscr{L}_2$-norm, $\|u\|_2=(\int_0^\infty u(s)^{\mathrm{T}}u(s)\,\mathrm{d}s)^{1/2}$. $\|\cdot\|_\infty$ denotes the H_∞ -norm. I_n is the unit matrix of order n, $0_{n,m}$ is the $n\times m$ zero matrix and $\mathrm{diag}\{A,B\}$ denotes a block diagonal matrix with A and B on the diagonal. $d^{(m)}(t)$ denotes the mth derivative of the vector function $d(t)$ and by $col\{a,b\}$ we denote the vector $[a^{\mathrm{T}}\ \ b^{\mathrm{T}}]^{\mathrm{T}}$.

2. Problem formulation

We consider the following linear, time-invariant, multi-input–multi-output (MIMO) system:

$$G(s)=\bar{D}(s)^{-1}\bar{N}(s)\mathrm{e}^{-s\tau}, \tag{1}$$

where $\bar{D}(s)=\sum_{k=0}^{l_1}\bar{D}_k s^k$ and $\bar{N}(s)=\sum_{k=0}^{l_1-1}\bar{N}_{l_1-1-k}s^k$ are $m\times m$ and $m\times r$ polynomial matrices, respectively, m is the dimension of output vector y and r is the dimension of the control input u to the plant. Time-delay τ is an uncertain constant time-delay in either the control input channel or in the measured output that, for a given scalar bound h, satisfies:

$$0\leqslant\tau\leqslant h. \tag{2}$$

We assume that $\bar{D}(s)$ is row reduced, namely that $\deg\det\{\bar{D}(s)\}=\sum_{i=1}^m l_i=\bar{n}$, where l_i is the degree of ith row of $\bar{D}(s)$ (it is assumed that $l_i\geqslant l_{i+1}$, $i=1,\ldots,m$) and $\bar{n}$ is the minimal order of $G(s)$ [3]. We denote by $\bar{D}_h$ the coefficient matrix of the highest order term in each row of $\bar{D}(s)$. To simplify our derivation in the sequel we assume that $\bar{D}_h$ does not incorporate uncertain parameters. Thus, we can assume, without loss of generality, that $\bar{D}_h=I_m$.

The system parameters are not completely known. We assume that they lie within a polytope

$$\Omega=Co\{\Omega_1,\Omega_2,\ldots,\Omega_{\bar{L}}\}, \tag{3}$$

where $\Omega_j=\begin{bmatrix}\bar{N}_{0,j} & \ldots & \bar{N}_{l_1-1,j}\\ \bar{D}_{0,i} & \ldots & \bar{D}_{l_1-1,j}\end{bmatrix}$, $j=1,\ldots,\bar{L}$, are the vertices of the polytope.

We seek a dynamic output-feedback controller whose transfer function matrix is given by the following left matrix fractional description:

$$H(s)=\bar{A}^{-1}(s)\bar{B}(s) \tag{4}$$

of an appropriate order.

In the development below we consider an additional finite energy disturbance vector $w(t)\in\Re^q$. We also consider an objective vector signal $z(t)\in\Re^p$ that will be defined below according to the performance requirements. We address the following problem:

The robust H_∞ control problem: For a prescribed scalar $\gamma>0$, find a controller that asymptotically stabilizes the system and satisfies

$$J_\infty=\int_0^\infty (z(s)^{\mathrm{T}}z(s)-\gamma^2 w(s)^{\mathrm{T}}w(s))\,\mathrm{d}s<0 \quad \forall w(t)\not\equiv 0\in\mathscr{L}_2 \tag{5}$$

over the entire uncertainty polytope and for all allowed delays τ.

We note that the initial conditions of the system is assumed to be zero.

Remark 1. The theory below can readily be extended to the case of different delays τ_i, $i=1,\ldots,q$ in the input channels, where we have the following plant model:

$$G(s)=\bar{D}(s)^{-1}\sum_{i=1}^{q}\bar{N}_i(s)\mathrm{e}^{-s\tau_i}.$$

For the sake of simplicity of the notations and in order to avoid matrices of large size, we treat below the case of $q=1$.

3. The augmented state–space model

It follows from (1) and the stated model assumptions that

$$\left(s^{l_1}I_m+\sum_{k=0}^{l_1-1}\bar{D}_k s^k\right)Y(s)=\left(\sum_{k=0}^{l_1-1}\bar{N}_{l_1-1-k}s^k\right)\mathrm{e}^{-s\tau}U(s). \tag{6}$$

Note, that the last j rows in $\bar{D}_i$ and $\bar{N}_{l_1-1-i}$, $0\leqslant i\leqslant l_1-l_{m-j+1}-1$, $j=1,\ldots,m-1$, are identically zero.

Choosing in (4):

$$\bar{A}(s)=s^{l_1-1}I_m+\sum_{k=0}^{l_1-2}\bar{A}_k s^k \quad\text{and}\quad \bar{B}(s)=\sum_{k=0}^{l_1-1}\bar{B}_{l_1-1-k}s^k \tag{7}$$

we obtain the following:

$$\left(s^{l_1-1}I_m+\sum_{k=0}^{l_1-2}\bar{A}_k s^k\right)U(s)=\left(\sum_{k=0}^{l_1-1}\bar{B}_{l_1-1-k}s^k\right)Y(s). \tag{8}$$

The configuration of the system and its controller is described in Fig. 1.

We define the following state vector $\xi(t)=col\{y^{(l_1-1)}(t),\ldots y(t),u^{(l_1-2)}(t),\ldots,u(t)\}$. Hence, we have the following state–space realization for the system of (1)–(4).

$$\dot{\xi}(t)=A_0\xi(t)+A_1\xi(t-\tau)+B_0\bar{u}(t)+B_1\bar{u}(t-\tau), \tag{9a}$$