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Modelling the capacitance of multi-layer conductor-facing interdigitated electrode structures

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ABSTRACT

Interdigitated electrode structures have applications in a myriad of fields and have become attractive for in-line electrochemical detection in lab-on-a-chip microsystems. Analytical models can replace complex and expensive numerical simulations to determine the capacitance of interdigitated electrode structures in cases where simple geometries can be assumed. Closed-form analytical expressions derived from Schwarz–Christoffel conformal mappings were used to determine the capacitance of multi-layer interdigitated electrode structures with an additional parallel continuous electrode. The partial capacitance approach was reformulated to find the capacitance of multi-layer structures with non-monotonically changing permittivities. The analytical model was found to be in close agreement with finite-element simulations. The model is used for optimization of an insulated transducer for sensitive detection of surface processes.

1. Introduction

Interdigitated electrode structures have been employed across many fields, from acoustic sensing in telecommunications to non-destructive testing [1]. The development of lab-on-a-chip devices and the associated need to downscale instrumentation for on-chip analyte detection and manipulation has driven the need for innovation of miniaturized transducers. Microfabricated interdigitated electrode structures are widely used in microfluidic systems for in-line analytical bioelectrochemistry [1–7] and dielectrophoretic particle control [8–12].

To design for high sensitivity of the electrochemical transducer, it is important to estimate the electrical coupling between the electrode fingers of the interdigitated multi-electrode array. While numerical models can provide an accurate representation of the physical, chemical, and electrodynamic characteristics of an electrochemical cell, they put high computational costs on the design and optimization process. Therefore, inexpensive analytical solutions that can estimate the capacitive coupling between the electrode fingers can be useful as a priori estimates.

Such closed-form analytical expressions that determine electrodynamic characteristics of coplanar electrode structures can be derived from conformal mapping methods [13]. For instance, these have been applied to determine the cell constants of planar-interdigitated conductivity sensors [14], for optimization of an electrolyte conductivity sensor [15], characterization of coplanar electrodes in microchannels [16,17], optimization for the detection of affinity binding of biomolecules [18], and to determine the electric field pattern of two-electrode structures in flow cytometer designs [19].

In many applications, interdigitated electrode fingers are embedded below multiple dielectric or conductive layers, that are assumed to be stratified with respect to the electrode plane. Igreja and Dias [20,21] demonstrated that conformal mappings and the partial capacitance method can be used to determine the capacitance and impedance of interdigital electrodes in multi-layered structures with semi-infinite domain boundaries above the electrodes. They discussed the effects of monotonically decreasing and increasing permittivity profiles of the dielectric layers above the interdigitated electrodes. They found that their results correlated considerably better with numerical finite-element simulations than models previously reported by Wu et al. [22] and Grevorgian [23].

The current research efforts have focused on the integration of interdigitated electrode structures into digital microfluidic (DMF) devices for in-line electrochemical detection. Comprehensive reviews of digital microfluidics can be found in [24–27]. When embedding interdigitated multi-electrode arrays within DMF devices, the domain boundary above the interdigitated electrode structure cannot be assumed at infinity, and in fact is terminated by a finite-domain continuous grounded top electrode. In addition, hydrophobic passivation layers that prevent fluid

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adhesion and electrode degradation result in non-monotonical permittivity profiles.

The works on analytical expressions for the electric- and magnetic field patterns in conductor-backed coplanar waveguides [28], as well as the effect of a ground plane on coplanar lines [29] both relate to the effect of an additional parallel ground electrode. However, these did not account for multiple (dielectric) layers between the coplanar electrodes and the additional parallel electrode nor did they take into consideration interdigitated electrode structures. Hence a model is needed to determine the capacitance of an interdigitated electrode structure facing a continuous top electrode with multiple dielectric and conductive layers in between. The model shall support preliminary characterization of such electrode structures prior to more in-depth and expensive numerical simulation techniques.

Presented here is an approach based on Schwarz–Christoffel conformal mappings (SCM) to find the capacitance of interdigitated multi-electrode arrays with an additional parallel continuous ground electrode. It is shown that that SCM-aided approach can be used for structures of the same type as discussed in [20,21]. In addition, an adaptation of the partial capacitance method for multi-layer structures is presented, and it is further suggested that it can be applied to structures in which the permittivity of the stratified layers above the electrode fingers is changing non-monotonically.

2. Methods

A sketch of a typical coplanar interdigitated electrode structure is shown in Fig. 1. The structure has two interlocked comb-like electrode finger arrays to create alternating tiers of electrodes connected to two electrical terminals ($\phi = +V$ and $\phi = -V$). The electrode fingers are assumed to have the same width w throughout and are separated by an inter-electrode gap g. The length L is sufficiently large, such that three-dimensional fringing field effects near the electrode finger ends are negligible.

The capacitance of a multi-electrode structure with any even number of electrodes N, where $N \geq 4$, can be determined from network analysis of concatenated individual unit cells [20],

$$C = (N-3)\frac{C_I}{2} + 2\frac{C_I C_E}{C_I + C_E} \tag{1}$$

where C_I is the capacitance of an interior unit cell, and C_E is the capacitance of an exterior unit cell (Fig. 1(b)). Accordingly, the capacitance of a two-electrode structure becomes $C = C_E/2$. How to determine unit cell capacitance is explained in coming paragraphs, and can be expressed in terms of two non-dimensional parameters: the metalization ratio, η [20], and the height-to-width ratio, r, defined by

$$\eta = \frac{w}{w+g} \tag{2}$$

and

$$r = \frac{2h}{w+g} \tag{3}$$

where h is the height of a layer above the electrode fingers. Moreover, the total capacitance including the layers below and above the interdigitated electrode structure is determined by adding the capacitance of the lower and upper half space.

2.1. The partial capacitance method

The interdigitated multi-electrode array is embedded below stratified layers, as shown in Fig. 2. The material is isotropic within each layer and only varies across layer interfaces in the vertical direction. The boundary conditions are specified according to the partial capacitance method and define the exterior and interior unit cells' geometry [20,21]. The partial capacitance method can be used to estimate the capacitance of a coplanar electrode structure embedded in a multi-layer assembly of dielectric materials with varying permittivity using either the parallel partial capacitance (PPC) or series partial capacitance (SPC) solution [30] based on [29,31,32,23,33].

The multi-layered interdigitated electrode structure in Fig. 2(a) applies to the PPC method noting that the permittivity is decreasing across each layer interface away from the electrode plane. The reduction in permittivity from one layer to the next is assumed to act as an electric field barrier. The layers are cascaded in a parallel-type configuration, and Neumann boundary conditions (i.e., magnetic walls with $\frac{\partial \phi}{\partial n} = 0$) are assumed at the layer interfaces. For the PPC method, the capacitance of a unit cell (interior or exterior) with n layers is

$$C_{cell} = \sum_{i=1}^{n-1} (\varepsilon_{r,i} - \varepsilon_{r,i+1}) C_i' + \varepsilon_{r,n} C_n' \tag{4}$$

where $\varepsilon_{r,i}$ is the relative permittivity of the ith layer, and C_i' is given by $C_i' = \varepsilon_0 L \kappa_{cell}^c(\eta, r_i)$. The cell constant κ_{cell}^c is defined by the cross-sectional geometry (2-D) and boundary conditions of the unit cell, ignoring three-dimensional fringing field effects near the ends of the electrode fingers. The superscript c indicates that the unit cells are considered as capacitive elements.[1]

For cases when the permittivity is monotonically increasing and the electric field is more strongly guided away from the electrode plane, the layers are assumed to be coupled in series, yielding the SPC method (Fig. 2(b)). It assumes Dirichlet boundary conditions (i.e., electric walls with $\phi = 0$) at the layer interfaces, and the total capacitance of the unit cell becomes

$$\frac{1}{C_{cell}} = \sum_{i=1}^{n-1} \left(\frac{1}{\varepsilon_{r,i}} - \frac{1}{\varepsilon_{r,i+1}} \right) \frac{1}{C_i'} + \frac{1}{\varepsilon_{r,n}} \frac{1}{C_n'} \tag{5}$$

Note that the numerical value of the Dirichlet boundary condition is obviously different in reality. Here, it serves the purpose of forcing the electric field lines be incident perpendicularly with the layer interfaces, whereas the Neumann boundary condition for the PPC approach mandates that the electric field lines are parallel to the layer interfaces.

The cell capacitance is hence specified by the permittivity and cell constant of the individual layers. Although not indicated explicitly, the cell constants in the PPC expressions are generally different than the cell constants in the SPC expressions, even though the height of the layers may be the same. This is because the magnitude of the cell constant is also determined by the choice of boundary conditions, i.e., Neumann or Dirichlet boundary conditions. Typically for open configurations the topmost layer is assumed to extend towards infinity, such that $C_n' = \varepsilon_0 L \kappa_{cell}^c(\eta, \infty)$, for both the PPC and SPC approach.

2.2. Reformulating the partial capacitance approach

Neither the PPC nor the SPC approach are applicable to instances with arbitrarily alternating permittivity across layer interfaces. The interdigitated electrode structure developed in the current work considers layers that result in non-monotonically varying permittivity. The partial capacitance approach can be reformulated such that it is possible to approximate the capacitance for any structure with arbitrarily varying permittivities. Eqs. (4) and (5) can be

[1] Typically the cell constant is given as the geometric proportionality factor relating the resistance and resistivity.