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Dissipativity and stabilization of nonlinear repetitive processes[☆]

Pavel Pakshin[a,b,*], Julia Emelianova[a], Mikhail Emelianov[a], Krzysztof Gałkowski[c], Eric Rogers[d]

[a] Arzamas Polytechnic Institute of R.E. Alekseev Nizhny Novgorod State Technical University, 19, Kalinina Street, Arzamas, 607227, Russia
[b] Lobachevsky State University of Nizhny Novgorod, Prospekt Gagarina, 23, 603950, Nizhny Novgorod, Russia
[c] Institute of Control and Computation Engineering, University of Zielona Góra, ul. Podgórna 50, 65-246 Zielona Góra, Poland
[d] Department of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, UK

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ABSTRACT

Repetitive processes are characterized by repeated executions of a task defined over a finite duration with resetting after each execution is complete. Also the output from any execution directly influences the output produced on the next execution. The repetitive process model structure arises in the modeling of physical processes and can also be used to effect in the control of other systems, the design of iterative learning control laws where experimental verification of designs has been reported. The existing systems theory for them is, in the main, linear model based. This paper considers nonlinear repetitive processes using a dissipative setting and develops a stabilizing control law with the required conditions expressed in terms of vector storage functions. This design is then extended to stabilization plus disturbance attenuation.

1. Introduction

The systems considered in this paper make a series of sweeps, also termed passes, through dynamics defined over a fixed finite duration termed the pass length [1]. On completion of each pass, the system resets to the starting location and the next pass can begin, either immediately after the resetting is complete or after a further period of time has elapsed. Each pass output is termed the pass profile and the distinguishing feature of these systems is that the pass profile on the previous pass acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. Such systems are known as repetitive processes.

Repetitive processes propagate information over a subset of the upper-right quadrant of the 2D plane and in this paper the notation for variables is of the form $y_k(t)$ where y is the scalar or vector valued variable, the nonnegative integer k denotes the pass number and $t \in [0, T]$ denotes the temporal variable defined over the finite duration pass length $T < \infty$, where the dynamics in the temporal variable can be discrete or differential. An industrial example described in [1], with references to the original modeling work, is long-wall coal cutting where the pass profile is the height of the stone/coal interface above some datum line and the objective is to extract the maximum amount of coal without penetrating the stone/coal boundary. The cutting machine rests on the most recently produced pass profile during the production of the next pass profile and therefore this is an industrial repetitive process. The unique control problem is that the sequence of pass profiles $\{y_k\}$ generated can contain oscillations that increase in amplitude from pass-to-pass, i.e., with k.

If these oscillations occur in a mining or other industrial example, solving a stabilization problem is the alternative to lost production resulting from a down time to enable their manual removal. This problem cannot, however, be solved using standard, or 1D, systems theory/algorithms since this approach ignores their inherent 2D systems structure, i.e., information propagation occurs from pass-to-pass and along a given pass. Also the initial conditions are reset before the start of each new pass.

The available results on the control of linear repetitive processes starts from a stability theory developed [1] using an abstract model of the dynamics in a Banach space setting that includes a very large number of processes with linear dynamics and

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* Corresponding author at: Arzamas Polytechnic Institute of R.E. Alekseev Nizhny Novgorod State Technical University, 19, Kalinina Street, Arzamas, 607227, Russia.
E-mail address: pakshin@apingtu.edu.ru (P. Pakshin).

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a constant pass length as special cases. The existence of this theory has also led to the emergence of problem areas where using a repetitive process setting for analysis has advantages. An example is classes of Iterative Learning Control (ILC) laws where experimental verification has been reported [2]. Another area is the analysis of OL-Nash games with a gas pipeline application [3].

If a linear model approximation of the dynamics is not possible then comparatively much less work has been reported on the stability of nonlinear 2D (or nD, $n > 2$) systems, see, e.g., [4,5] and references therein. The continual emergence of new possible application areas, such as ILC applied to bead morphology in laser metal deposition processes [6], adds further support to the development of this general area, starting from a stability theory and leading on to control law design.

It is important to place repetitive processes in context and, in particular, explain why they cannot be controlled by standard theory. These processes differ from repetitive control, see, e.g., [7], where the reference signal is periodic and there is no stoppage time between the end of one cycle and the start of the next one. Repetitive processes do have a resetting after each pass is complete and moreover the structure of the boundary conditions is critical to their stability and hence control properties.

In the simplest case, the initial state on each pass is independent of the previous pass dynamics but in some applications this assumption is too simplistic and it is necessary to have state initial conditions that are an explicit function of the dynamics produced on the previous pass. Such conditions arise in the long wall coal cutting example, see [1] for a detailed explanation that cites the original work in this area. Examples exist [1] where ignoring the resetting and thereby converting the dynamics into an 'equivalent' infinite pass length standard system gives incorrect conclusions as does modeling the state initial conditions as pass independent when they are actually pass dependent. Pass dependent boundary conditions also arise in the use of repetitive process stability theory to study iterative solution algorithms for nonlinear dynamic optimal control problems based on the maximum principle [8]. Hence there is a need in both theory and applications for a rigorous stability theory for these processes that extends to control law design.

In the case of 1D nonlinear systems, dissipativity theory [9] is one of the most powerful methods for control design, where a particular form, known as passivity (and its generalizations) see, e.g., [10] plays a significant role in solving the global feedback stabilization problem for a wide class of systems. This paper gives new results on a dissipativity approach to the stabilization of discrete and differential nonlinear repetitive processes, where the term 'differential' refers to dynamics along the pass governed by a matrix differential equation for which the term continuous could equally be used. The results are obtained using a vector storage function approach that is different from that in [11,12] and results in a control law to guarantee exponential stability of the controlled process. This design is then extended to stabilization plus disturbance attenuation.

2. Dissipativity and the stabilization of discrete repetitive processes

2.1. Process description and definitions

The discrete nonlinear repetitive processes considered in this section are described by the following state–space model over $0 \le p \le T-1$, $k \ge 0$,

$$\begin{aligned} x_{k+1}(p+1) &= f_1(x_{k+1}(p), y_k(p), u_{k+1}(p)),\\ y_{k+1}(p) &= f_2(x_{k+1}(p), y_k(p), u_{k+1}(p)), \end{aligned} \tag{1}$$

where the integer $T < \infty$ denotes the number of samples over the pass length and on pass k, $x_k(p) \in \mathbb{R}^{n_x}$ is the current pass state vector, $y_k(p) \in \mathbb{R}^{n_y}$ is the pass profile vector, f_1, and f_2 are nonlinear functions such that $f_1(0,0,0) = 0$, $f_2(0,0,0) = 0$. This last requirement is necessary to have zero as the equilibrium state. The boundary conditions, i.e., the pass state initial vector sequence and the initial pass profile, are assumed to be of the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \quad k \ge 0,\\ y_0(p) &= f(p), \quad 0 \le p \le T-1, \end{aligned} \tag{2}$$

where the entries in $d_{k+1} \in \mathbb{R}^{n_x}$ are known constants and the entries in $f(p) \in \mathbb{R}^{n_y}$ are known functions of p. It is assumed that d_{k+1} and $f(p)$ have bounded energy, i.e., there exist finite real numbers $M_f > 0$, $\kappa_d > 0$ and $0 < \lambda_d < 1$ such that d_{k+1} and $f(p)$ satisfy

$$|f(p)|^2 \le M_f, \qquad |d_{k+1}|^2 \le \kappa_d \lambda_d^{k+1}, \quad k \ge 0, \tag{3}$$

where $|q|$ denotes the Euclidean norm of a vector q.

The scalar $|M_f|^{\frac{1}{2}}$ is an upper bound on the norm of the initial pass profile and $\kappa_d^{\frac{1}{2}}$ is an upper bound on the initial pass state vector sequence, which is assumed to be bounded in norm by an exponentially decreasing sequence with rate of convergence (in k) λ_d. In particular, λ_d represents the rate of convergence in k of the pass initial state vector sequence. From this point onwards, all references to the boundary conditions for the processes considered will assume that they satisfy (3). Moreover, in this paper the state initial vector on each pass is independent of the previous pass profile vector but, as discussed in the previous section, there are applications where this assumption is too strong. Sufficient progress with the case considered in this paper should prompt further research on this more general case.

In the control and systems theory developed for linear repetitive processes, the stability along the pass property has formed the basis for control law design and experimental verification [1,2]. This property demands that a bounded initial pass profile produces a bounded sequence of pass profiles for all possible values of the pass length and is based on linear operator theory in a Banach space setting. Hence it cannot be directly transferred to the nonlinear case.

Stability along the pass requires that the sequence (in k) of pass profiles and state vectors are bounded independent of the pass length. In the case of nonlinear discrete nonlinear repetitive processes stability should also enforce boundedness (in k) independent of the pass length of the sequences (in k) of pass profiles and state vectors and one possible approach would be to use a Lyapunov function approach as in the stability analysis of 1D nonlinear systems.

The Lyapunov approach is based on properties of the function itself and for discrete dynamics of its increments, but the dynamics of repetitive processes are determined by the state vector x and the pass profile vector y, which are functions of the two independent variables p and k. A candidate Lyapunov function for these processes can be chosen as a scalar function, $V(x, y)$, but to construct the gradient along the trajectories of (1) it is required to have $x_{k+1}(p) - x_k(p)$ and $y_k(p+1) - y_k(p)$ as functions of x and y. These quantities can only be found by solving (1) but then all of the advantages of the Lyapunov approach are lost.

A powerful method in the analysis and control of 1D systems is dissipativity theory [9], especially the particular case of passivity theory [9,10] where an extension of a Lyapunov function termed a storage function is used. Previous work [13,14] developed a stability theory for discrete nonlinear repetitive processes based on the use of vector Lyapunov functions and the discrete counterpart of their divergence along the trajectories to characterize the property of exponential stability. In this paper the problem considered is stabilization and disturbance attenuation using a