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Controllability for transmission wave/plate equations on Riemannian manifolds

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ABSTRACT

In this paper we consider controllability for transmission system of coupling wave equations with Euler–Bernoulli equations on Riemannian manifolds. We show that such a system is exactly controllable by boundary controls only along the exterior boundary, which means there is no control on the interface of the transmission system. Our proofs rely on the Hilbert Uniqueness Method (HUM) and geometric multiplier method.

1. Introduction

Transmission systems appear in many practical control systems such as chemical reactions coupling, structural-acoustic systems and many other interactive physical processes. Among these systems are the transmission problem of hyperbolic systems, which attract much attention and have strong physical backgrounds, for example, it can describe the displacement of flexible structures consisting of two physically different types of materials. The analysis of controllability and stabilization for the transmission of hyperbolic systems has been widely carried out.

In this article we aim to discuss the controllability of transmission system of coupling wave equations with Euler–Bernoulli equations on Riemannian manifolds, of which system we considered the stabilization property in our recent work [1]. More precisely, we consider the case where we only introduce the control on the exterior boundary. That means there is no control on the interface of the transmission problem. We prove that the system is exactly controllable under some geometric assumptions on the interface. We found that such kind of geometric assumption cannot be removed. The reason is that, the system may not be controllable only from exterior boundaries, due to total reflection at the interface, as already pointed out by many other authors, see [2,3].

1.1. Statement of the problem and the main result

Let M be a complete two dimensional Riemannian manifold of class C^3 with C^3-metric $g(\cdot,\cdot) = \langle\cdot,\cdot\rangle$. For each $x \in M$, M_x is the tangential space of M at x. Denote the set of all n order tensor fields on M by $T^n(M) = \cup_{x\in M} T_x^n(M)$, where n is a nonnegative integer. It is well known that the space $T_x^n(M)$ of n order tensor on M_x is an inner product space. Its inner product is defined by

$$\langle T_1, T_2\rangle_{T_x^n} = \sum_{i_1,i_2,\ldots,i_n=1}^{2} T_1(e_{i_1},\ldots,e_{i_n}) T_2(e_{i_1},\ldots,e_{i_n}) \quad \text{at } x, \tag{1.1}$$

for any $T_1, T_2 \in T_x^n(M)$, where $e_1,\ e_2$ is an orthonormal basis of M_x for $x \in M$. For any $T \in T^2(M)$, the trace of T is defined by

$$trT = \sum_{i=1}^{2} T(e_i, e_i). \tag{1.2}$$

We denote by ∇ the gradient, by D the Levi-Civita connection and by $\Delta = \mathrm{div}(\nabla)$ the Laplace–Beltrami operator in the Riemannian metric g. For any vector field H on M, DH is the covariant differential of H which is a second order tensor field in the following sense:

$$DH(X,Y) = D_Y H(X) = \langle D_Y H, X\rangle \quad \text{for all } X, Y \in M_x,\ x \in M. \tag{1.3}$$

For scalar function u we have $Du = \nabla u$.

Let Ω be an open, bounded, connected subset of M with smooth boundary Γ such that $\bar\Omega = \bar\Omega_1 \cup \bar\Omega_2$, where Ω_i, $i = 1, 2$ are two disjoint open connected bounded domains with smooth boundary. They satisfy that $\bar\Omega_1 \cap \bar\Omega_2 = S$, $\partial\Omega_1 = S \cup \Gamma_1$ and $\partial\Omega_2 = S \cup \Gamma_2$ (see Fig. 1).

We consider the wave equation in Ω_1 coupled with the Euler–Bernoulli plate equation in Ω_2 by the interface S with boundary controls $\phi(x,t)$, $\psi(x,t)$, $\zeta(x,t)$ on $\Gamma_0 \subset \Gamma$ which is a

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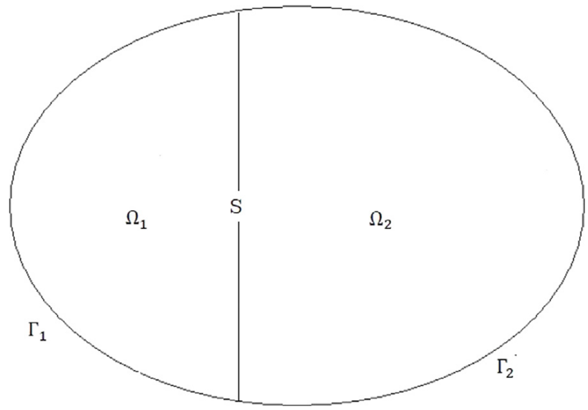

Fig. 1. The domain.

subset of the exterior boundary Γ. More precisely, we consider the following system:

$$\begin{cases} \partial_t^2 u_1(x,t) - \Delta u_1(x,t) = 0, \quad \text{in } \Omega_1 \times (0,+\infty), \\ \partial_t^2 u_2(x,t) + \Delta^2 u_2(x,t) - (1-\mu)\mathrm{div}(\mathcal{K}\nabla u_2)(x,t) = 0, \\ \quad \text{in } \Omega_2 \times (0,+\infty), \\ u_i(x,0) = u_i^0(x), \qquad \partial_t u_i(x,0) = u_i^1(x), \\ \quad \text{in } \Omega_i, i = 1,2, \\ u_1 = u_2, \qquad B_1 u_2 = 0, \qquad B_2 u_2 = \partial_{\nu_1} u_1, \\ \quad \text{on } S \times (0,+\infty), \\ u_1 = 0, \quad \text{on } (\Gamma_1/\Gamma_0) \times (0,+\infty), \\ u_1 = \phi, \quad \text{on } (\Gamma_1 \cap \Gamma_0) \times (0,+\infty), \\ u_2 = \partial_{\nu_2} u_2 = 0, \quad \text{on } (\Gamma_2/\Gamma_0) \times (0,+\infty), \\ u_2 = \psi, \quad \text{on } (\Gamma_2 \cap \Gamma_0) \times (0,+\infty), \\ \partial_{\nu_2} u_2 = \zeta, \quad \text{on } (\Gamma_2 \cap \Gamma_0) \times (0,+\infty), \end{cases} \tag{1.4}$$

where $\mathcal{K}$ is the Gaussian curvature function on Ω_2. Here $\nu_i = \nu_i(x)$ denotes the unit outward normal vector of Ω_i along $\partial\Omega_i = \Gamma_i \cup S$ for $i = 1,2$. In the above system, $0 < \mu < \frac{1}{2}$ is the Poisson coefficient, and the boundary operators B_1, B_2 are defined on $\partial\Omega_2 = \Gamma_2 \cup S$ as follows:

$$B_1 y = \Delta y - (1-\mu)D^2 y(\tau_2,\tau_2),$$
$$B_2 y = \partial_{\nu_2}\Delta y + (1-\mu)\partial_{\tau_2}\left(D^2 y(\tau_2,\nu_2)\right) + \mathcal{K}\partial_{\nu_2} y,$$

where $D^2 y$ is the Hessian of y and τ_2 is the unit tangential vector along the boundary $\partial\Omega_2 = \Gamma_2 \cup S$.

Remark 1.1. The term $(1-\mu)\mathrm{div}(\mathcal{K}\nabla u_2)$ in the system (1.4) comes from the curvedness of the Riemannian metric g. For details, see [4, Model, pp. 150].

To obtain the controllability of the problem (1.4), the following geometrical hypotheses are assumed:

Geometrical assumptions Given the triple $\{\Omega_1, S, \Omega_2\}$, there exists a vector field H on Riemannian manifold (M,g) such that the following three properties hold true:

(A.1) $DH(\cdot,\cdot)$ is strictly positive definite on $\overline{\Omega}$: there exists a constant $\rho > 0$ such that for all $x \in \overline{\Omega}$, for all $X \in M_x$ (the tangent space at x):

$$DH(X,X) \equiv \langle D_X H, X\rangle \geq \rho|X|^2. \tag{1.5}$$

(A.2) The control area $\Gamma_0 \triangleq \Gamma_0^1 \cup \Gamma_0^2$ satisfies

$$\Gamma_0^i = \{x \in \Gamma_i \big| \langle H, \nu_i\rangle > 0 \text{ on } \Gamma_i, i = 1,2\}. \tag{1.6}$$

(A.3) The interface satisfies

$$\langle H, \nu_i\rangle = 0, \quad \text{on } S. \tag{1.7}$$

Remark 1.2. For any Riemannian manifold M, the existence of such a vector field H in (A.1) has been proved in [5], where some examples are given, too. See also [4]. In the framework of Euclidean metric, one can take the vector field $H = x - x_0$. This was given first in the paper [6] as well as in [7,8]. Thus $DH(X,X) = |X|^2$ follows, which means assumption (A.1) always holds true with $\rho = 1$ for the Euclidean case.

Here we state our main result.

Theorem 1.1. *We assume the geometrical assumptions* (A.1), (A.2) *and* (A.3) *hold true. Then the transmission problem* (1.4) *is exactly* $L^2(\Omega_1) \times L^2(\Omega_2) \times H^{-1}(\Omega_1) \times H^{-2}(\Omega_2)$ *controllable by* $L^2(0,T;L^2(\Gamma_0^1)) \times L^2(0,T;L^2(\Gamma_0^2)) \times L^2(0,T;L^2(\Gamma_0^2))$ *controls, i.e., there exist some* $T > T_0$, *control functions* $\phi \in L^2(0,T;L^2(\Gamma_0^1))$, $\psi \in L^2(0,T;L^2(\Gamma_0^2))$ *and* $\zeta \in L^2(0,T;L^2(\Gamma_0^2))$ *such that the corresponding solution* $(u_1,u_2,\partial_t u_1,\partial_t u_2)$ *of* (1.4) *satisfies*

$$(u_1,u_2)(\cdot,T) = 0, \qquad (\partial_t u_1, \partial_t u_2)(\cdot,T) = 0,$$

where Γ_0^i *are given in* (1.6) *and*

$$T_0 = \frac{8}{\rho}\sup_{x\in\Omega_1}|H|. \tag{1.8}$$

1.2. Literature

Controllability for transmission problems were studied by several authors, for example, for the coupled wave equations with constant coefficients, the controllability results are presented in [2] by applying the Hilbert Uniqueness Method (HUM). For the coupled wave equations with variable coefficients (describing the wave with variable propagation speed), the boundary controllability is treated in [9], which involves the research for transmission problem of anisotropic elastic materials. The exact controllability for transmission plate equations was addressed in [10]. We refer to [11–15] for related results on transmission problems of other hyperbolic systems.

These works mentioned above offer fruitful results regarding the transmission systems in the framework of Euclidean metric. However, the problems on the compact Riemannian manifolds have limited results compared with the problems on Euclidean spaces. The main difficulty comes from the fact that the trapped geodesics in the general Riemannian manifolds can preclude the effectiveness of the canonical multiplier which plays an important role in obtaining the global estimates in the controllability analysis. At the same time, the transmission problems on Riemannian manifolds have strong practical applications. For example, once the plates in [16,17] have curved middle surfaces Ω, the transmission system considered by [16,17] becomes the one on the general Riemannian manifold (Ω, g), where g is the induced Riemannian metric.

In the present paper we consider the controllability of a coupled wave–plate system on Riemannian manifolds. The main tool we use is the geometric multiplier method, which first appeared in [5] and subsequently in [18–21,1], and many others. First, we establish multiplier equality for the coupled wave–plate system on Riemannian manifolds. Then, under the geometric assumptions on the domain, we obtain the controllability of the coupled system in Theorem 1.1.

The content of this paper is organized as follows. In Section 2, we establish the multiplier equality for the dual transmission system by the geometric multiplier method. Finally in Section 3, we present the proofs of the main results.