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On the stability analysis of phasor and classic extremum seeking control

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ABSTRACT

In this work we present a semi-global practical asymptotic stability analysis for phasor extremum seeking control with a general non-linear dynamic system. With the same technique applied to the classic band pass filter algorithm, we present a more relaxed (less constrained) semi-global practical asymptotic stability condition compared to earlier work. The results are based on a non approximated averaging for both control techniques.

1. Introduction

Extremum seeking control (ESC) is an on-line single objective optimization concept for dynamic plants. It is a model-less control technique. The purpose of the ESC controller is to adjust the plant input in order to achieve a maximal (minimal) value in the output for any given initial conditions of the plant, given that neither the optimal value of the input nor the optimal value of the output are known. Fig. 1 shows a typical maximization problem where the process is represented by its steady-state relation from input θ to output y. The task of the ESC is to adjust the input in order to reach the maximum output y^*.

Many applications of extremum seeking control can be found in the literature, for example braking system control, autonomous vehicles and mobile robots, yield optimization in bio-processing, etc. [1].

Many types of extremum seeking control were presented in the literature but the method of sinusoidal perturbation is preferred because it permits a faster convergence to the extremum on a time-scale comparable to that of the plant dynamics, which is an advantage over the numerically based methods that need the plant dynamics to settle down before optimization [2,3].

The classic ESC method (Fig. 2) adopts band pass filtering in which a periodic perturbation signal (usually a sinusoidal) with amplitude a and radian frequency ω is added to the input of the plant. Then a high pass filter (HPF), a multiplier, and a low pass filter (LPF) are used to produce an estimate ξ of the rate of change of output with respect to input (i.e. the gradient). Finally, an integrator with gain k is used to regulate ξ.

Stability analysis of ESC has received a lot of attention in the last two decades. Krstić and Wang [4] presented what can be considered as the most valuable stability analysis of extremum seeking control for the standard filters based approach for a general type of plants. Averaging and singular perturbation analysis were used to prove local stability of the classic ESC controller.

The concept of practical asymptotic stability is used in certain non-linear perturbed systems that do not have an equilibrium point, instead the system will converge to a small ball around the origin. The size of the region of attraction and the convergence ball depends mainly on the parameters of the perturbation in the system [5,6]. Tan et al. [7] presented the next step in the stability analysis of ESC. They adopt the practical stability concept from [6] and prove semi-global practical asymptotic stability of ESC. In their work, stability analysis was provided for different schemes such as ESC without filters, with only low pass filter and with both low pass and high pass filters.

In previous work, special assumptions were made on the steady-state map relating input and output, e.g. that this map has a single maximum (or minimum) either locally as in [4] or globally as in [7]. Also, the derivative of this map should equal zero only at the extremum point. In [8], performance of ESC was examined under different dither and modulation signals. In [7] it was noted that

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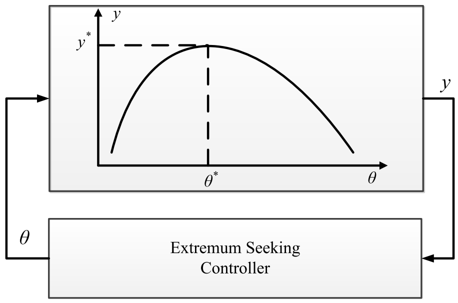

Fig. 1. Basic block of ESC controller.

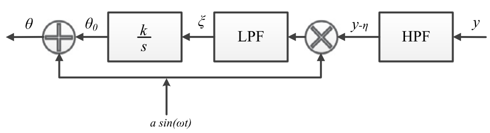

Fig. 2. Classic band pass based ESC controller.

the amplitude of the dither signal can overcome a local maximum (minimum). Later in [9], the existence of local extrema in the static map and their influence on the convergence of the system were studied and a time-varying amplitude of the dither signal was proposed in order to overcome the local minimum. The amplitude is initialized large and will decrease with time to reduce the convergence error that is proportional to amplitude of the signal.

Another perturbation based method called Phasor ESC was presented in [10,11] which is based on the estimation of the phasor of the plant output (or equivalently, the phase and amplitude of the first harmonic in the output of the plant). Then the optimal operation can be achieved through regulating the phasor.

The basic idea of phasor ESC is to interpret the output of plant into three components: a constant component, a sine component, and a cosine component.

$$y \approx \beta_0 + \alpha_1 \sin(\omega t) + \beta_1 \cos(\omega t). \tag{1}$$

A continuous time Kalman filter is employed to estimate the state vector $\mathbf{z} = [\beta_0, \alpha_1, \beta_1]^T$. Under the assumption that the Kalman filter has a state covariance matrix $Q = qI$ and output covariance r, the Kalman filter is simplified into a variable gain state estimator by finding a closed from solution of the special case periodic Riccati equation (see Proposition 1 in [10]). The Kalman gain

$$L(t) = \sqrt{\frac{q}{r}} \begin{bmatrix} 1 \\ \sqrt{2}\sin(\omega t + \zeta) \\ \sqrt{2}\cos(\omega t + \zeta) \end{bmatrix} \tag{2}$$

where ζ is a function of $\eta = \omega\sqrt{\frac{r}{q}}$ in the form $\zeta = 2\tan^{-1}(\varphi)$ where φ is the positive real root of

$$(\sqrt{8} - 1)\varphi^4 + 4\sqrt{2}\,\eta\,\varphi^3 + 6\varphi^2 + 4\sqrt{2}\,\eta\,\varphi - \sqrt{8} - 1 = 0. \tag{3}$$

Fig. 3 shows the simplified block diagram of the phasor ESC. The local stability analysis was presented in [10].

We would like to emphasize that this assumption is only used as a basis for an intuitive motivation of the proposed method. For the stability analysis, both in [10] and in the present manuscript, no assumption of the sinusoidal nature of the process output is made, instead a general, nonlinear state-space model of the process is assumed.

The main advantages of phasor ESC are the possibility of using higher perturbation frequency compared to the classic approach which enables faster convergence and also the possibility of use in a high noise environment [11].

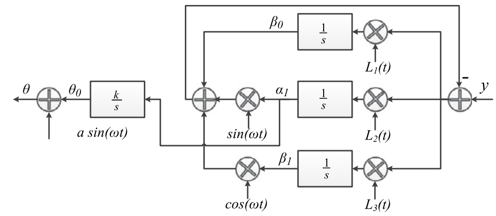

Fig. 3. The simplified phasor ESC.

In this work we will provide a proof of semi-global practical stability for the phasor extremum seeking feedback for a general type of systems. Also we provide a more relaxed semi-global practical stability proof for the classical band pass filter extremum seeking control (compared to [7]) in the sense that stability is achieved without limiting the amplitude of the perturbation signal and that the second derivative (and higher derivatives) of the static map are allowed to be zero at the global maximum (minimum). We also allow the first derivative to be zero on bounded intervals.

In Section 2 we introduce some definitions and preliminary results. A formulation of the general problem will be presented in Section 3. In Section 4 the main findings of the article will be shown in terms of conditions for practical stability of classical and Phasor based ESC. Finally, conclusions and future work are presented in Section 5.

2. Preliminaries

A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It belongs to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$ [12]. A continuous function $\sigma : [0, \infty) \to [0, \infty)$ is said to be a class $\mathcal{L}$ function if it is strictly decreasing and $\sigma(s) \to 0$ for $s \to \infty$. The function σ is called the convergence function [6].

A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if for each fixed s, the function $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r, and for each fixed r, the function $\beta(r, s)$ belongs to class $\mathcal{L}$ with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$ [12].

A function $f(x)$ is called quasi concave or unimodal [13,14] if there is a value $x = x^*$, where the function is increasing for $x < x^*$ and decreasing for $x > x^*$ and it is called strictly quasi concave if the function is strictly increasing for $x < x^*$ and strictly decreasing for $x > x^*$.

In general, periodic and perturbed dynamic systems may not have an equilibrium point and accordingly it is difficult to study their stability. Therefore, averaging may be applied to the system in order to remove the oscillation or perturbation so that stability analysis can be applied on the averaged system [12].

Definition 1 (*[6]*). The function $f(\mathbf{x}, t)$ is said to have an average $f_{av}(\mathbf{x})$ if there exists a class $\mathcal{KL}$ function β, a class $\mathcal{L}$ function σ and a $T^* > 0$ such that from every $T \geq T^*$ and from all $t \geq 0$

$$\left\| \frac{1}{T} \int_t^{t+T} f(\mathbf{x}, \tau)d\tau - f_{av}(\mathbf{x}) \right\| \leq \beta(|\mathbf{x}|, T) + \sigma(T). \tag{4}$$

Remark 1. Definition 1, which is for a general system (not necessarily periodic) differs from the Khalil definition [12] by the extra term $\beta(|\mathbf{x}|, T)$ which allows more flexibility in the averaging error with respect to the distance from the equilibrium (i.e. $|\mathbf{x}|$). The two definitions are equivalent when $T \to \infty$.

Semi global practical stability is a concept for describing systems that depend on small parameters and\or have some oscillatory behavior [6]. The following definition is essentially from [7].